Moment methods
and
nuclear level densities

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supported by a SSAA grant from US DOE-NNSA

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Computational Nuclear Structure SDSU

Tools:
mean-fields and moments

<u>3 approaches to nuclear level density:</u>

modified Fermi gas

Monte Carlo shell model

spectral distribution methods

mean-field (centroids or first moments)

residual interaction (spreading widths or second moments)

collective interaction (third moments)

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Fermi Gas Models

Single-particle energies from Hartree-Fock *mean-field*: $\varepsilon_i^{p,n}$

Single-particle density of states: $$g(\varepsilon) = \sum_i \delta(\varepsilon - \varepsilon_i)$$

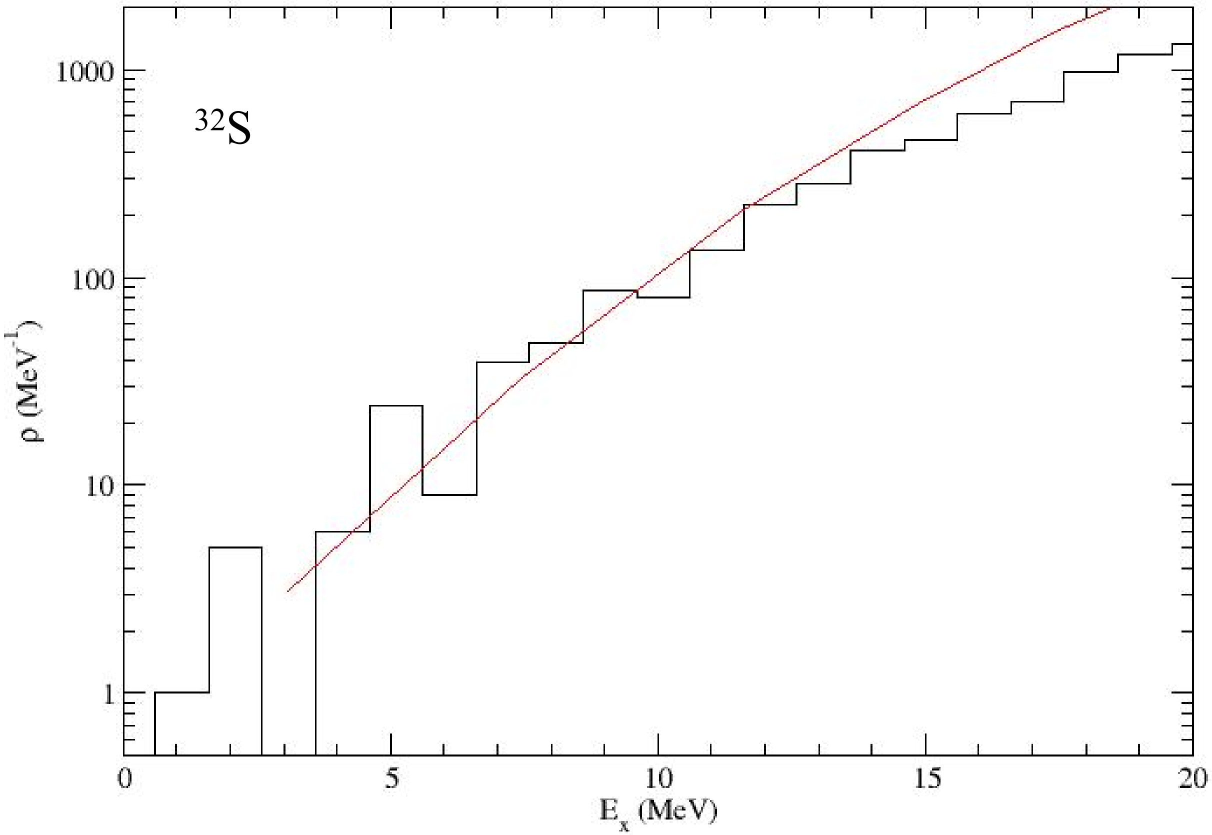
Partition function : $$\ln Z(\alpha, \beta) = \sum_i \ln\left(1 + \exp(\alpha - \beta\varepsilon_i)\right)$$

Then apply saddle-point method...

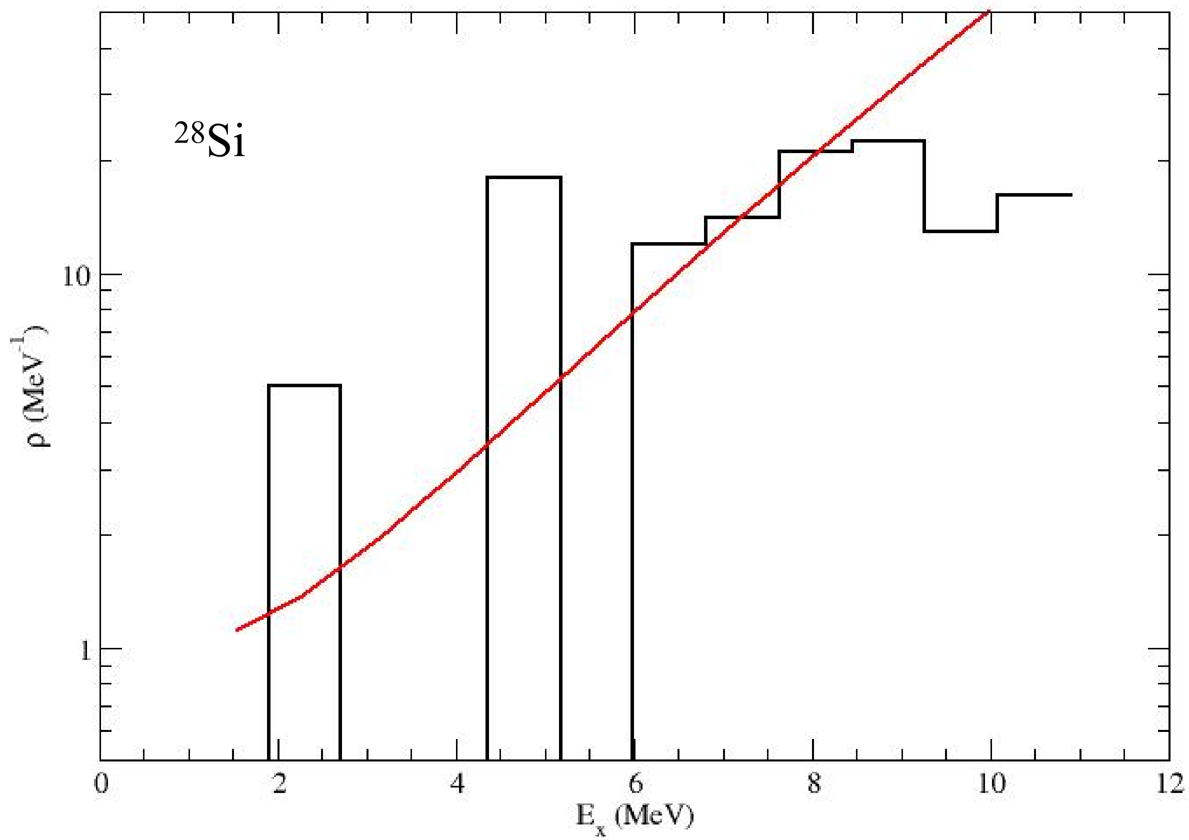
Please keep in mind the following is **deliberately crude** and **unsophisticated** and is meant as **motivation,** not **criticism** of this approach

Mean-field Level densities

First, start with “exact” level density from shell-model diagonalization and add in model of level density from mean-field calculation

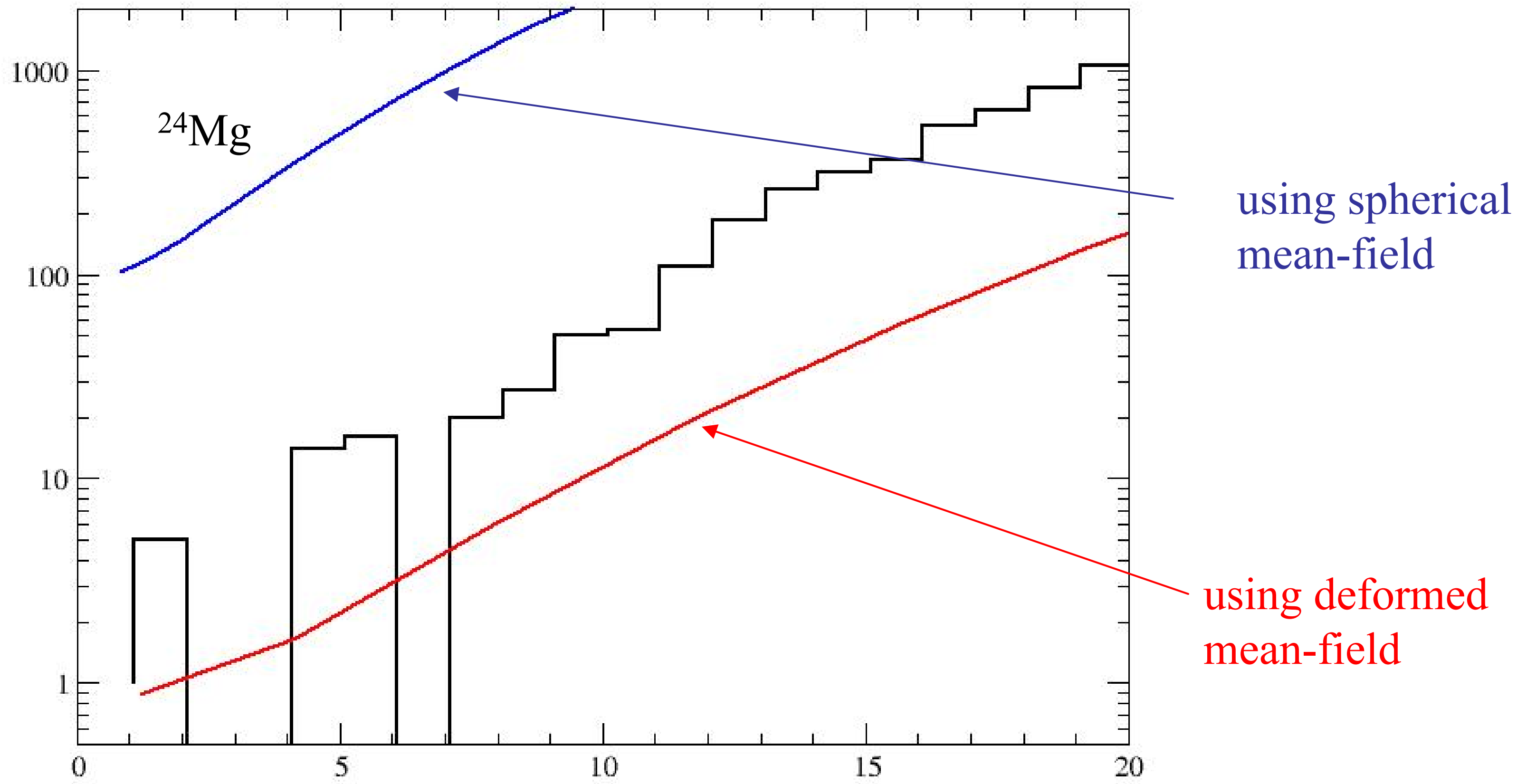


Mean-field Level densities

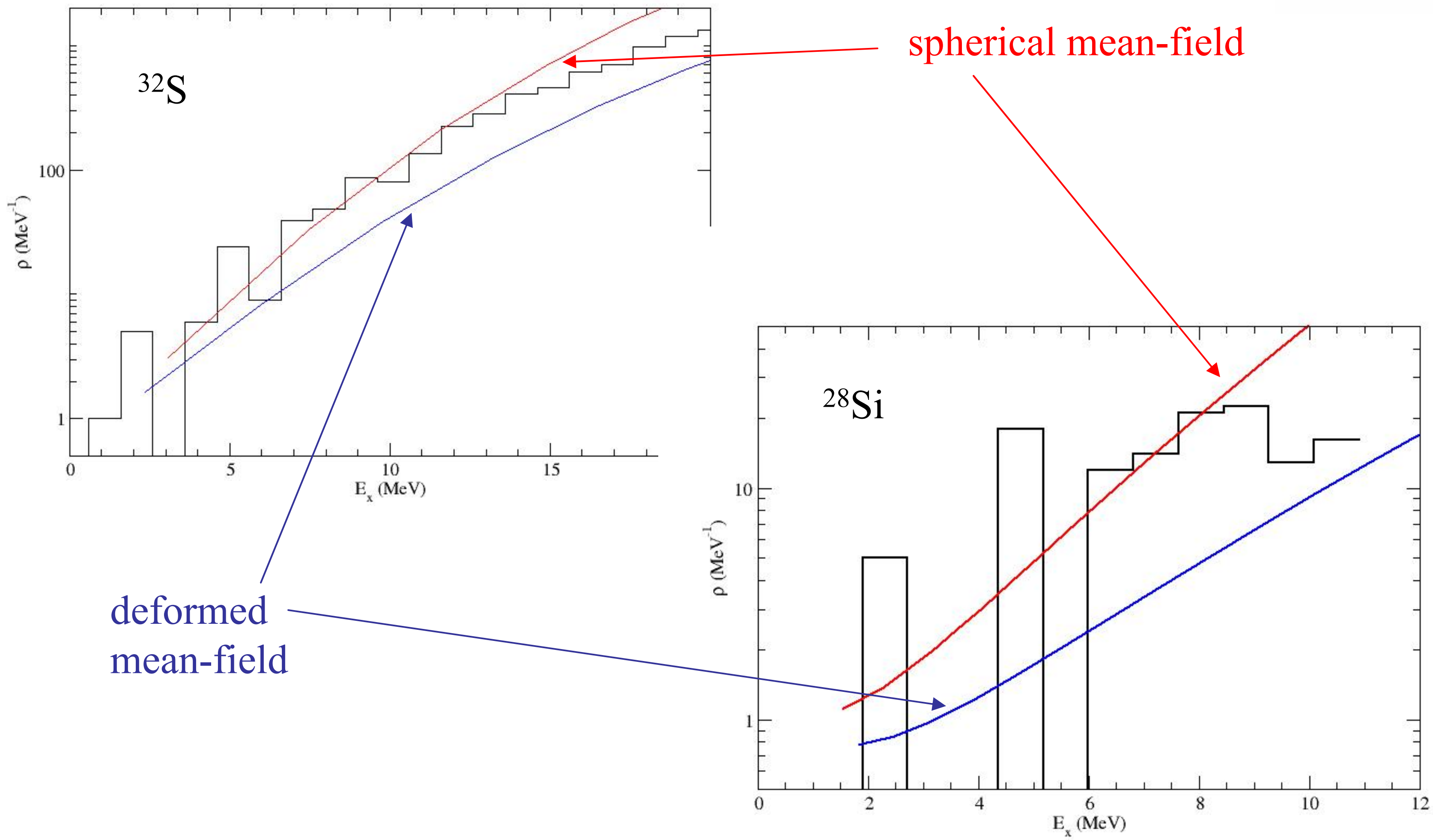


These are both spherical nuclei... what about deformed nuclides?

Mean-field Level densities



Mean-field Level densities



Mean-field Level densities

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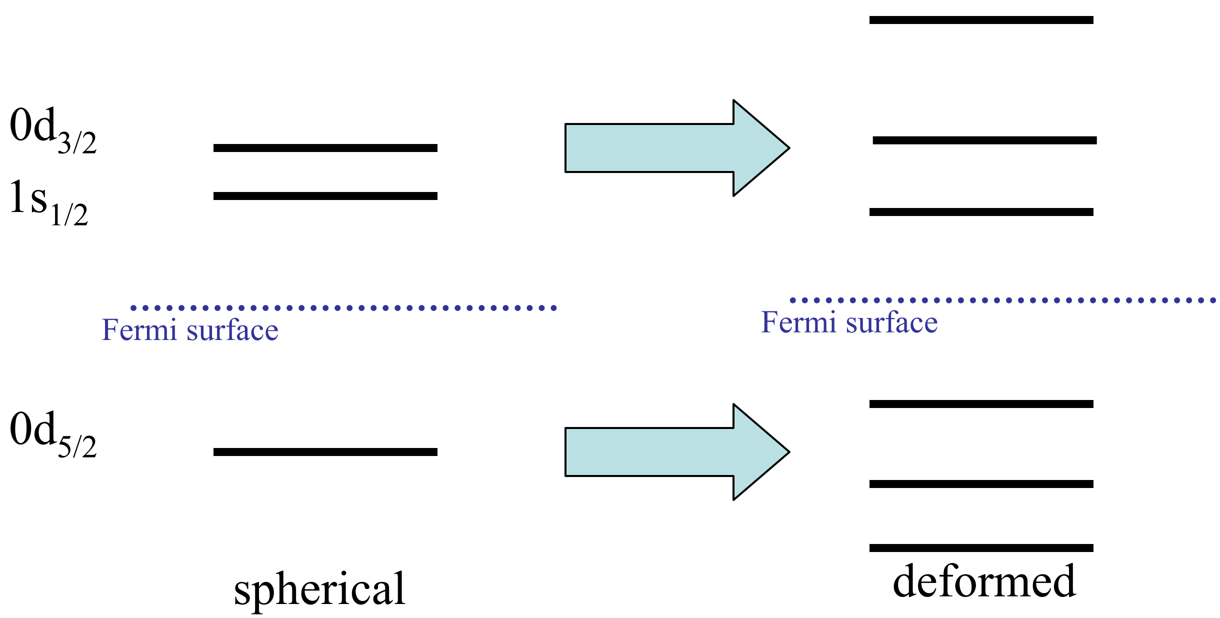
Difference is due to **fragmentation** of Hartree-Fock single-particle energies in deformed mean-field

This in turn is a manifestation of the residual interaction



Introduction to Statistical Spectroscopy

(also known as "spectral distribution theory")

Pioneered by J. Bruce French 1960's-1980's

other luminaries include: J. P. Draayer, J. Ginocchio, ***S. Grimes****, V. Kota, S.S.M. Wong, A.P. Zuker + many others...*

Problem: diagonalization is too hard and gives too much detailed information

Solution: instead of diagonalizing **H**, find moments: tr $\mathbf{H}^n$

Key question: how many moments do we need?

Rather than many moments (over the entire space) tr $\mathbf{H}^n$, n = 1,2,3,4,5,6,7...
compute low moments (n = 1,2,3,4) on *subspaces*

How we do it:
a detailed version

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The important configuration moments

Dimension $d_\alpha = Tr\mathbf{P}_\alpha$

Centroid: $\bar{E}_\alpha = \frac{1}{d_\alpha} Tr\mathbf{P}_\alpha \mathbf{H}$ Width: $\sigma_\alpha = \frac{1}{d_\alpha} Tr\mathbf{P}_\alpha \left(\mathbf{H} - \bar{E}_\alpha\right)^2$

Higher central moments $\mu_n(\alpha) = \frac{1}{d_\alpha} Tr\mathbf{P}_\alpha \left(\mathbf{H} - \bar{E}_\alpha\right)^n$

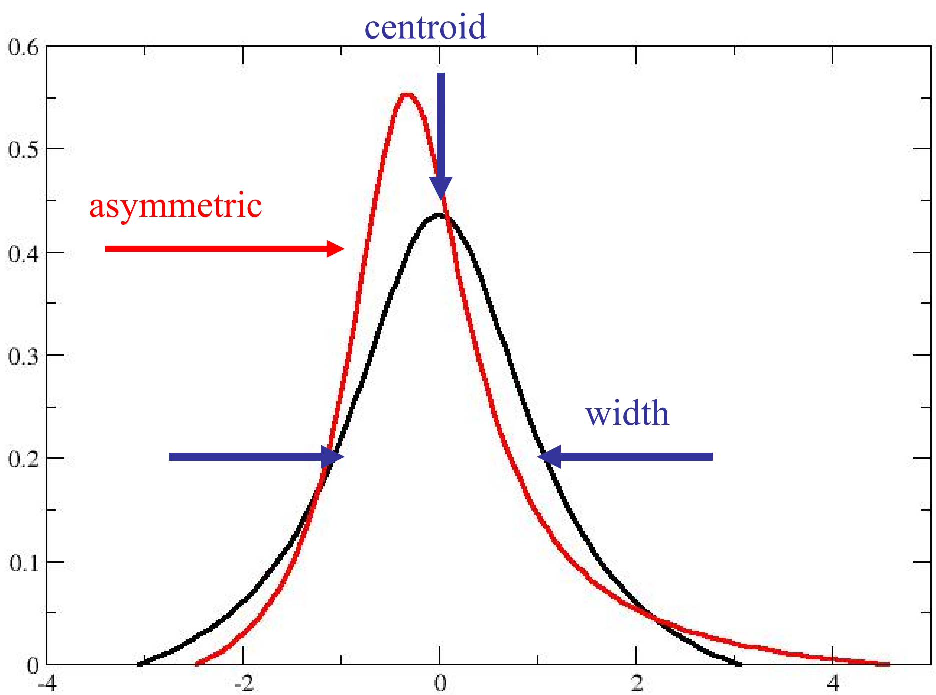
Scaled moments $m_n(\alpha) = \mu_n(\alpha) / \left(\sigma_\alpha\right)^n$

Asymmetry (or skewness): $m_3(\alpha)$

Excess : $m_4(\alpha)$ - 3 = 0 for Gaussian

Introduction to Statistical Spectroscopy

Primer on moments



Interpretation of moments:

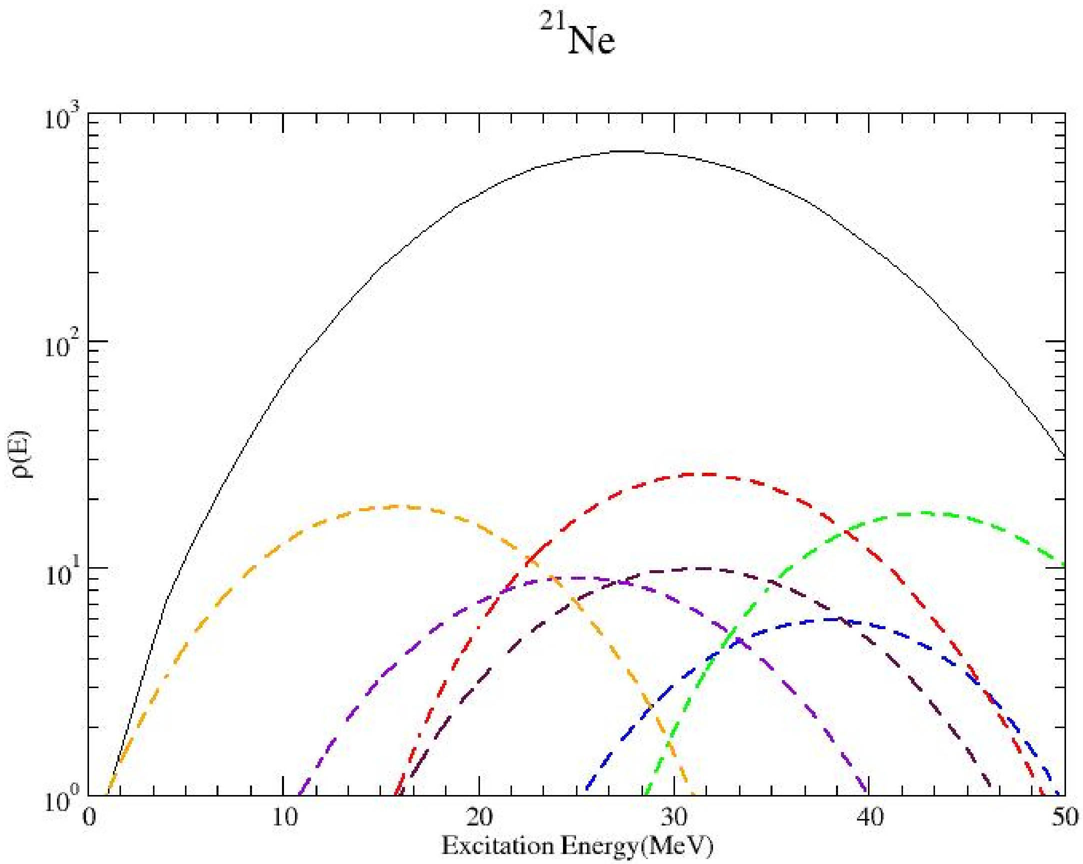
centroid = spherical HF energy

width = avg spreading width of residual interaction

asymmetry = measure of collectivity

Introduction to Statistical Spectroscopy

Then we consider the level density as being the sum of individual configuration densities



Level densities as a sum of configuration densities

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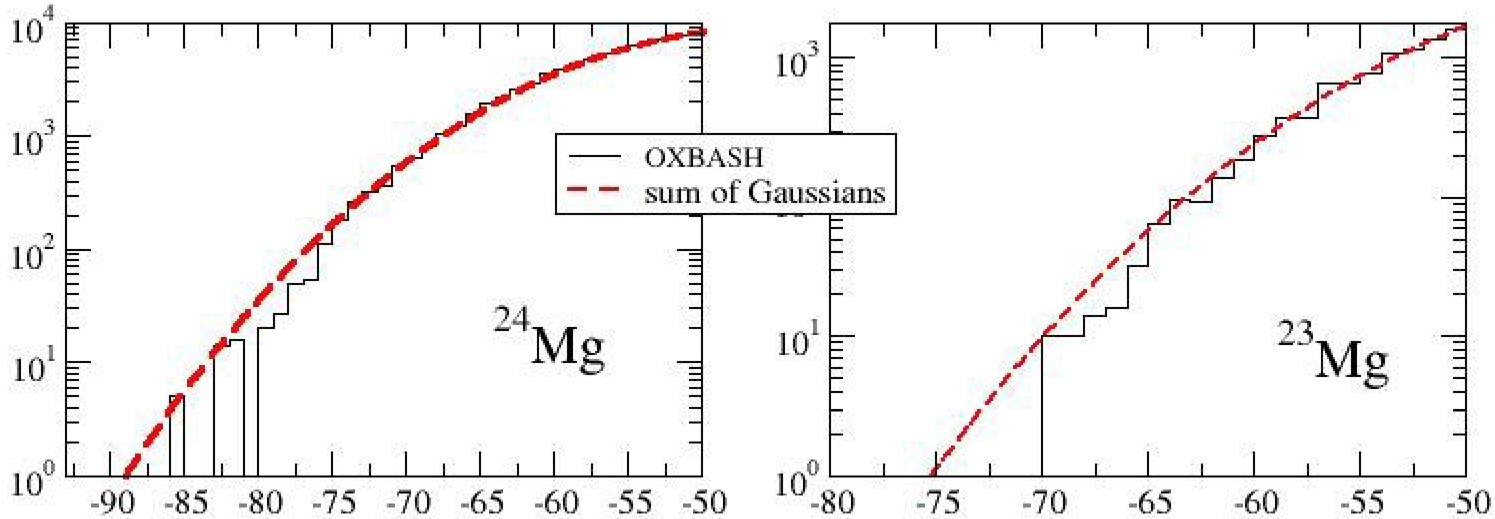
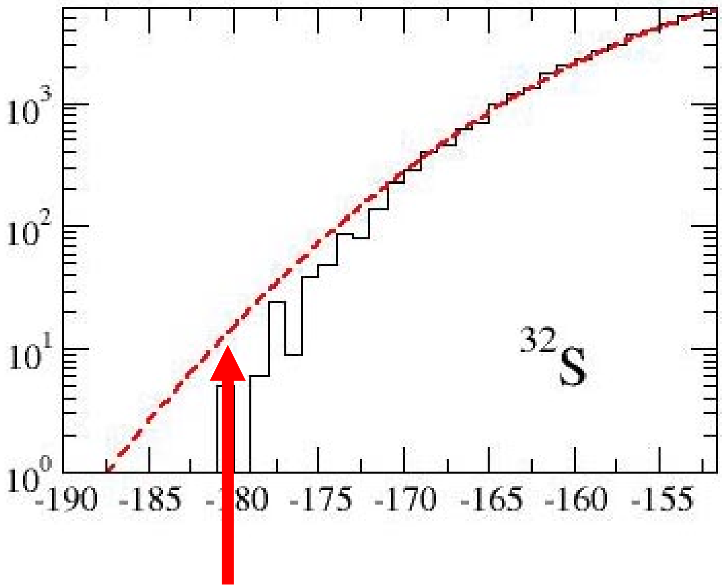
We model the level density as a sum of partial (configuration) densities, each of which are modeled as Gaussians



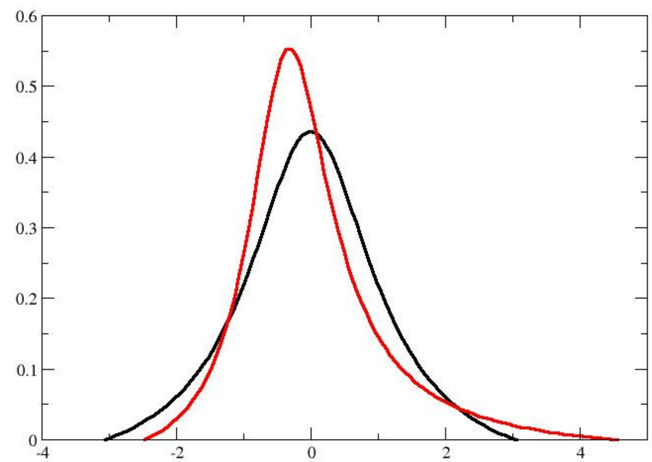
Level densities as a sum of configuration densities



Not satisfactory!

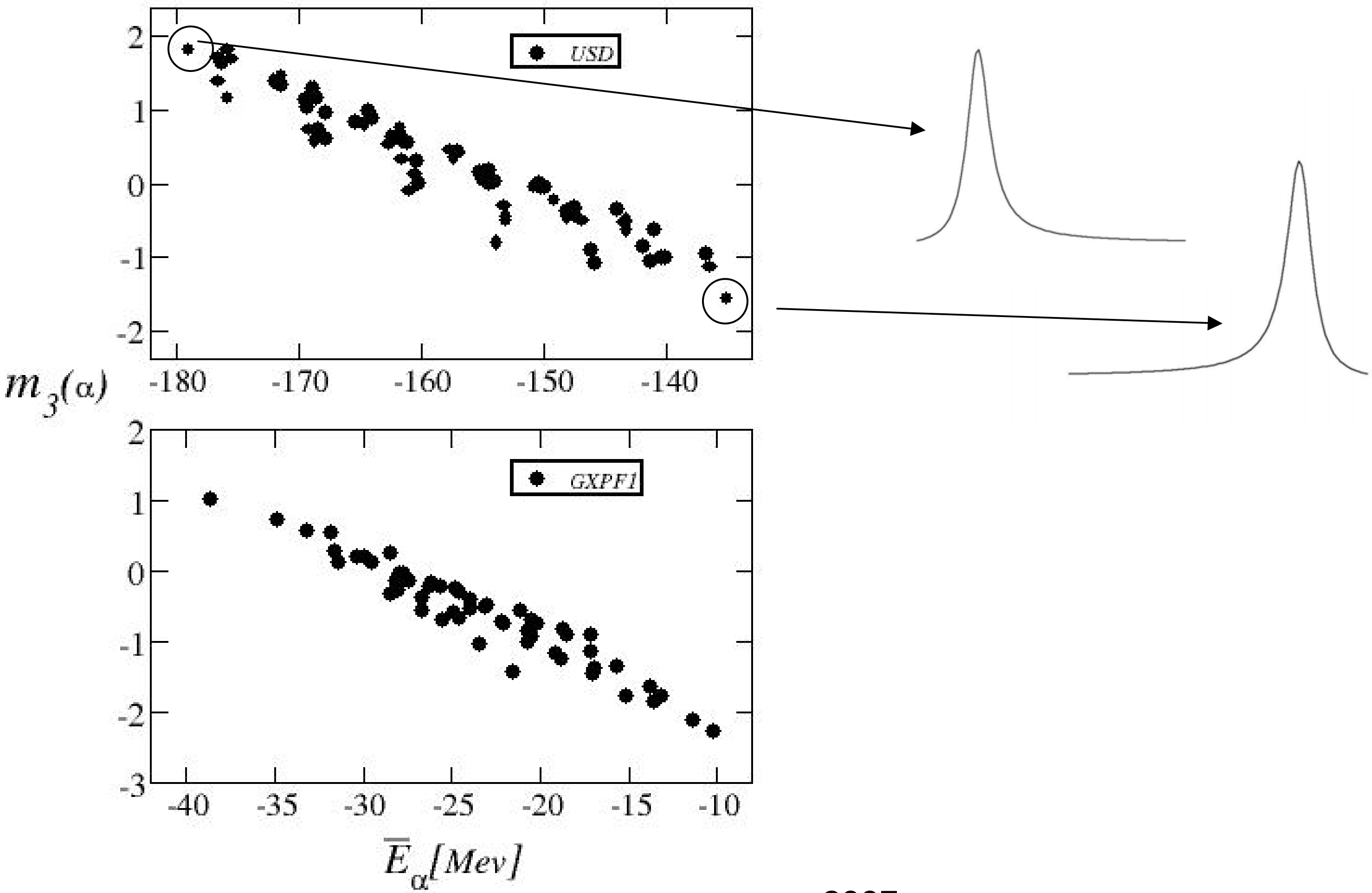
What can we do to improve our model?

Go to third moments: asymmetries



Shell-Model
Configuration moments

The configuration asymmetry varies almost linearly with the centroid



Shell-Model
Configuration moments

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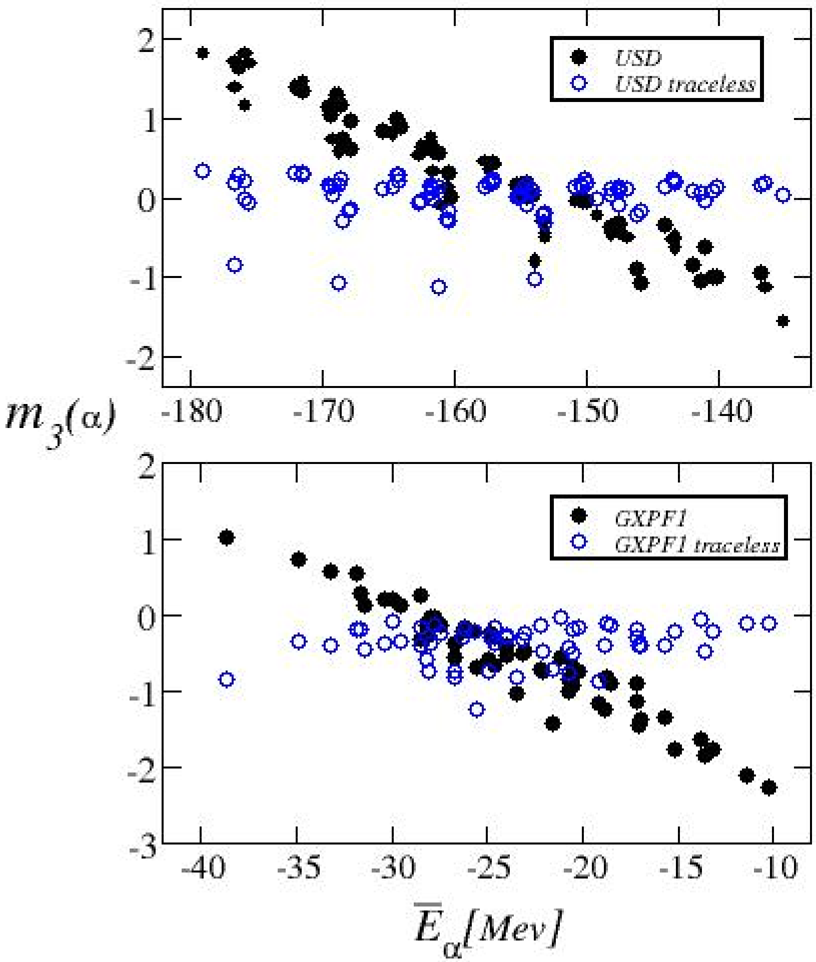

We can make all the centroids =0 by setting the "monopole" part of the interaction = 0
(this is called a "traceless" interaction in the vocabulary of statistical spectroscopy)

The monopole potential is related to the mean field!

Deviations from this trend are associated with strong collectivity

Collectivity and Asymmetries

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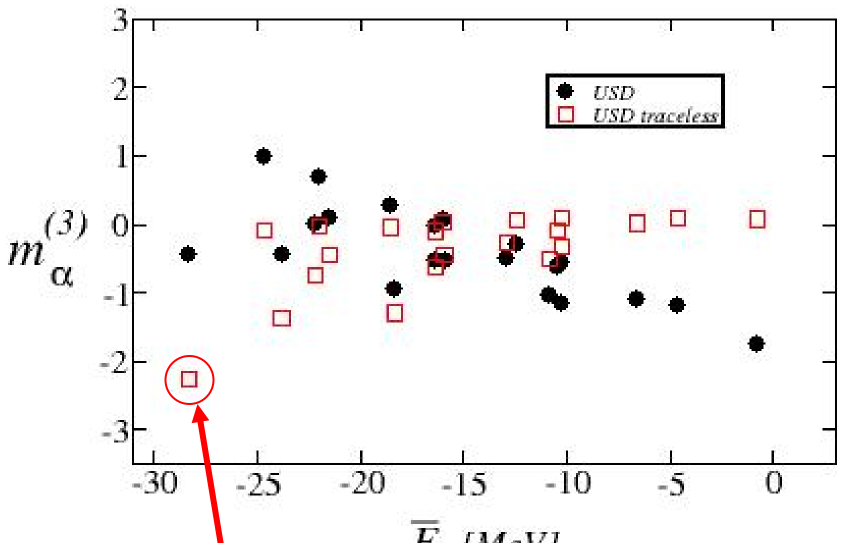
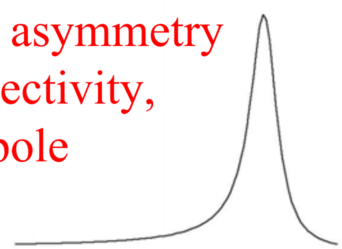
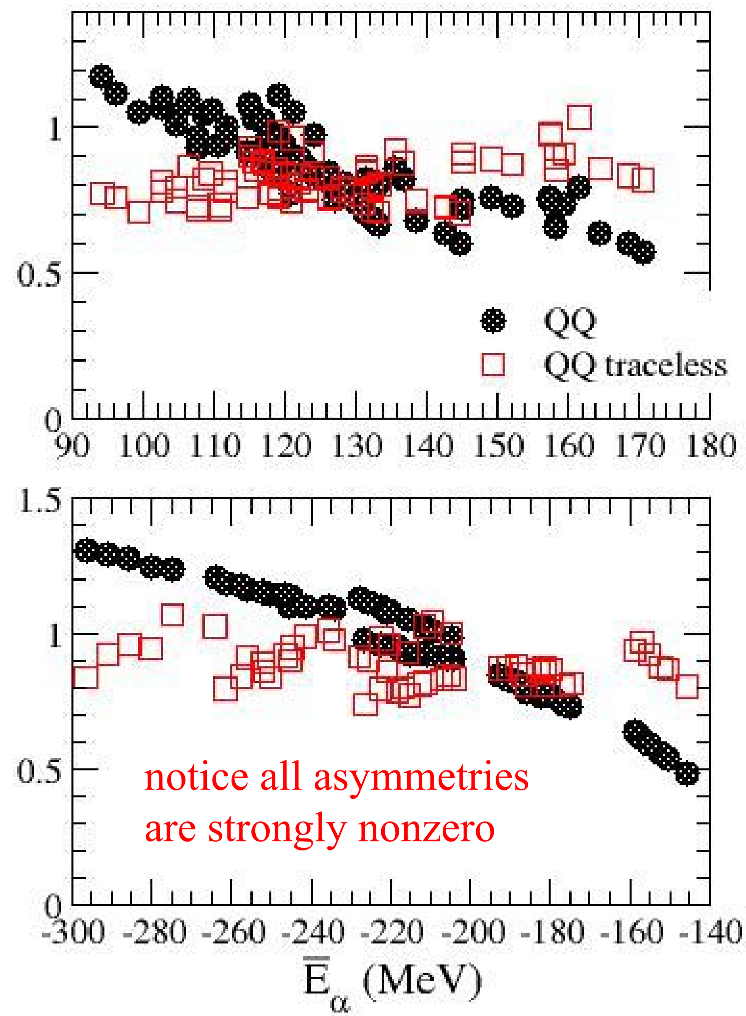


strong traceless asymmetry arises from collectivity, such as quadrupole





Level densities as a sum of configuration densities

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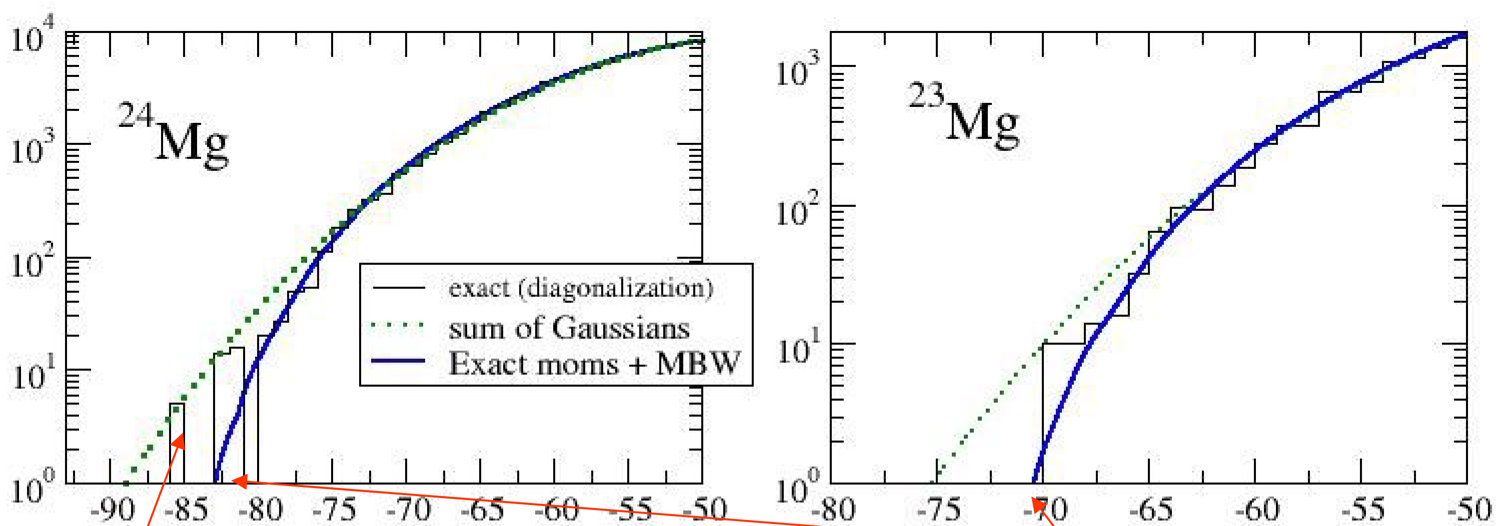
It is (often) important to include 3rd and 4th moments

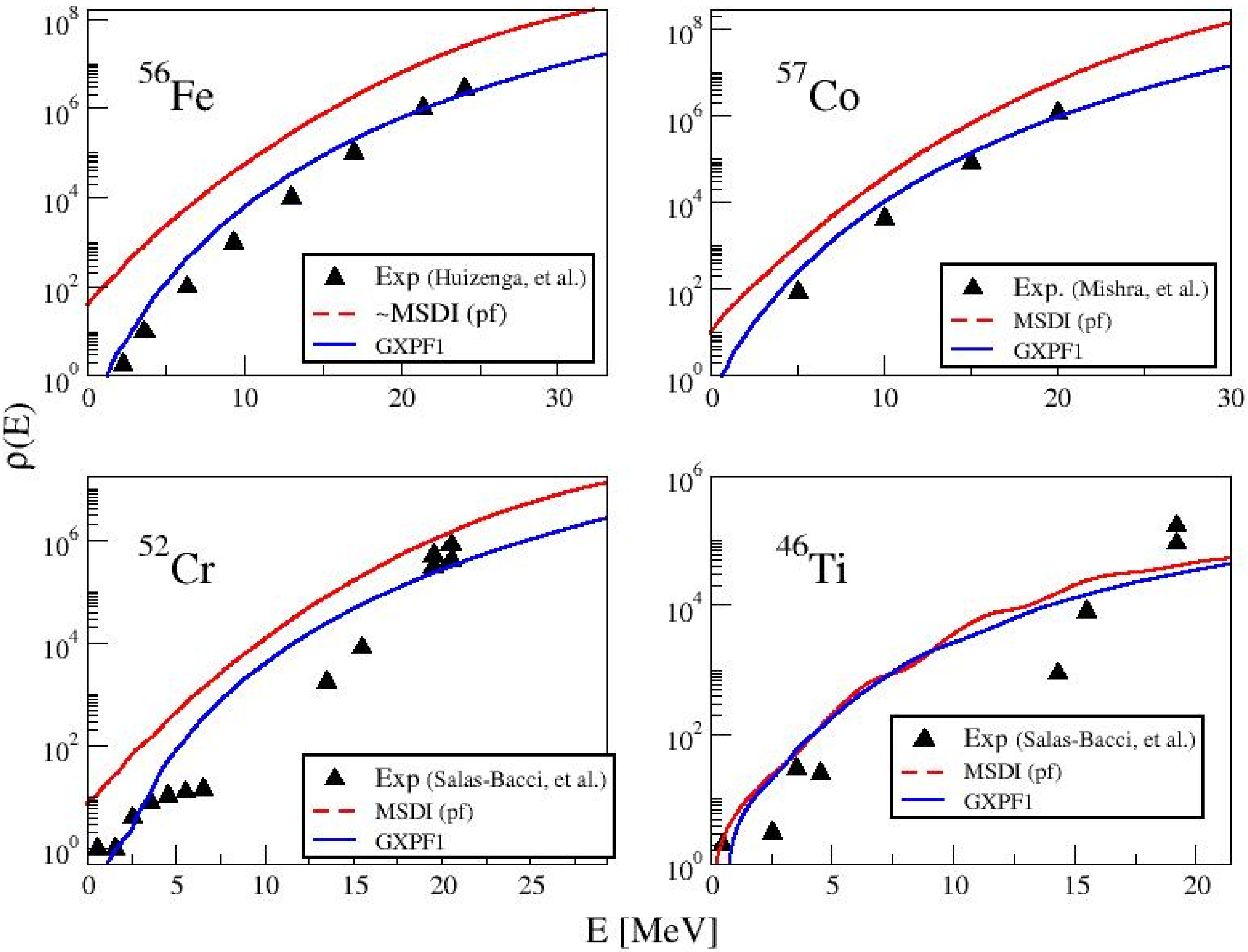
much better than using only second moments



collective states difficult to get

"starting energy" also difficult to control

Comparison with experiments



NB: computed
+ parity states
and multiplied × 2

Obligatory Summary

View nuclear many-body Hamiltonian through lens of moment methods:

1st (configuration) moments = mean-field

2nd moments = spreading widths of residual interaction

3rd moments = collectivity of residual interaction