Experimental Overview of Compound Nuclear Resonance Reactions

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The study of compound nuclear reactions is a vast and diverse field – here we focus on resonance reactions. We briefly summarize efforts at addressing the major issues: A – How to measure the resonances, B – How to categorize resonances (spin, parity, resonance energy and strength), C – How to describe the distribution of resonances strengths and spacings, D – How to assess data quality. Sample illustrative examples are provided for each of these topics.

COMPOUND NUCLEUS

A --ISOLATED RESONANCES

B -- OVERLAPPING RESONANCES

C -- CONTINUUM

FOCUS TODAY ON ISOLATED RESONANCES

REGION B IS MYSTERY TO ME

REGION C

HAUSER-FESHBACH

MANY EXPERTS ARE HERE

THEORY AND EXPERIMENT

QUASI-STATIONARY STATES AT HIGH EXCITATION LED TO BOHR AND COMPOUND NUCLEUS

RESONANCES VERY IMPORTANT FOR REACTORS, STEWARDSHIP, ASTROPHYSICS…

THEREFORE ISSUES

HOW TO MEASURE RESONANCES

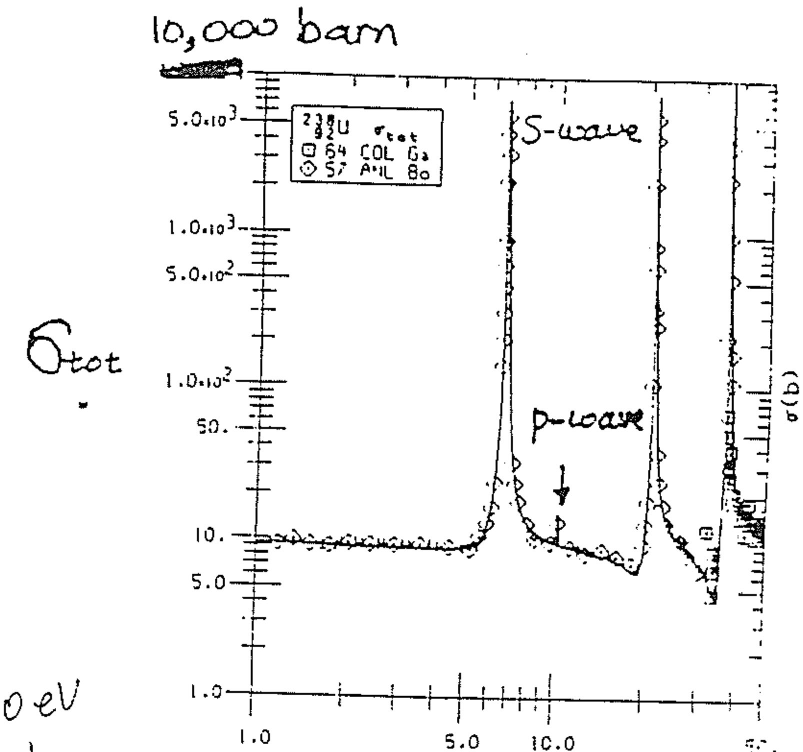
HOW TO CATEGORIZE RESONANCES (RESONANCE SPECTROSCOPY)

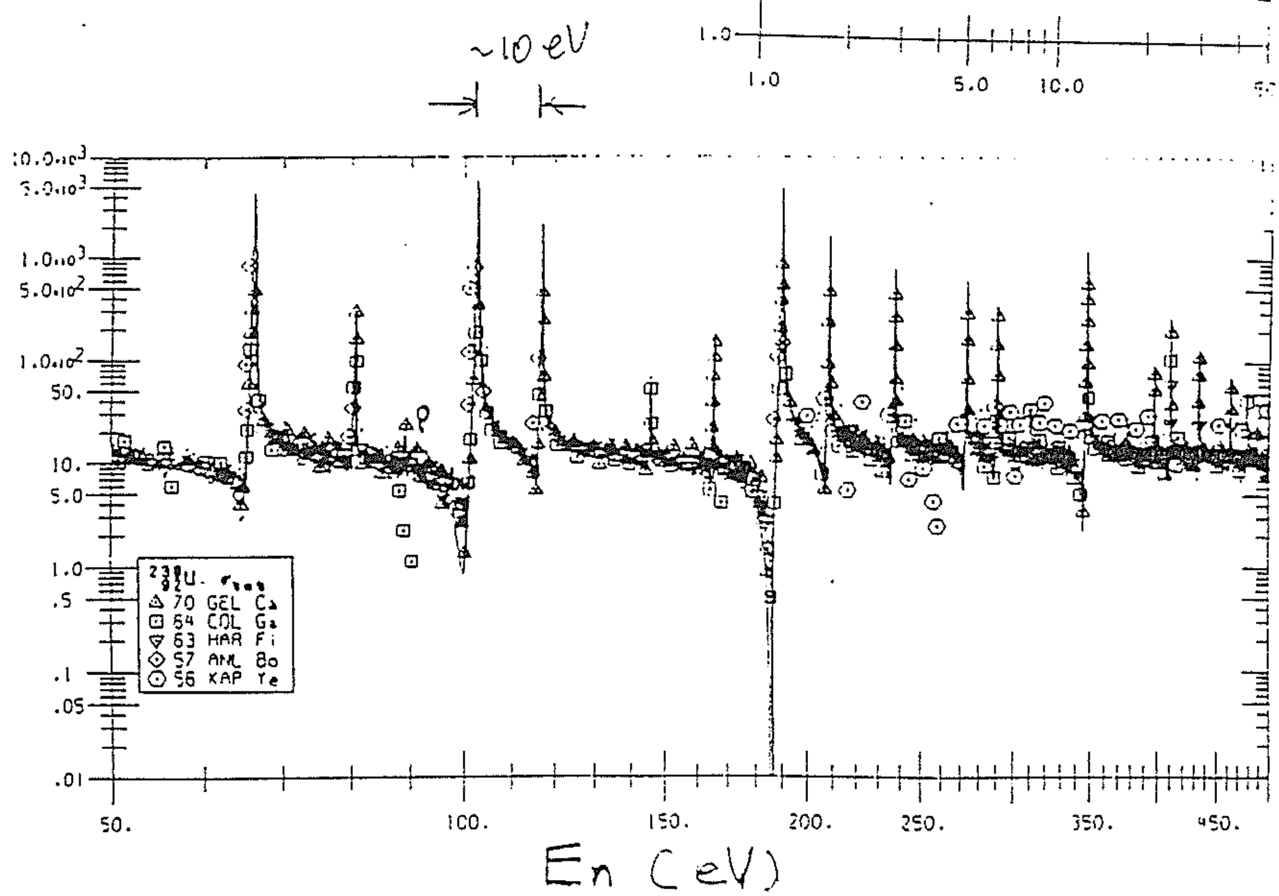
HOW TO DESCRIBE DISTRIBUTION OF RESONANCE STRENGTHS AND SPACINGS

HOW TO ASSESS DATA QUALITY

- Epithermal (0.1 – 10^5 eV) Neutron–Nucleus scattering:

$n + {}^{238}U$





NEED LOTS OF NEUTRONS
PLUS ENERGY RESOLUTION

USE ELECTRON MACHINES

ORELA

RPI

GEEL (IRMM)

SAROV

SPALLATION NEUTRON SOURCES

USE HIGH ENERGY PROTONS
CREATE LOTS OF NEUTRONS
MODERATE
USE TIME OF FLIGHT

HISTORICAL
NEVIS SYNCHROCYLOTRON
RAINWATER AND HAVENS

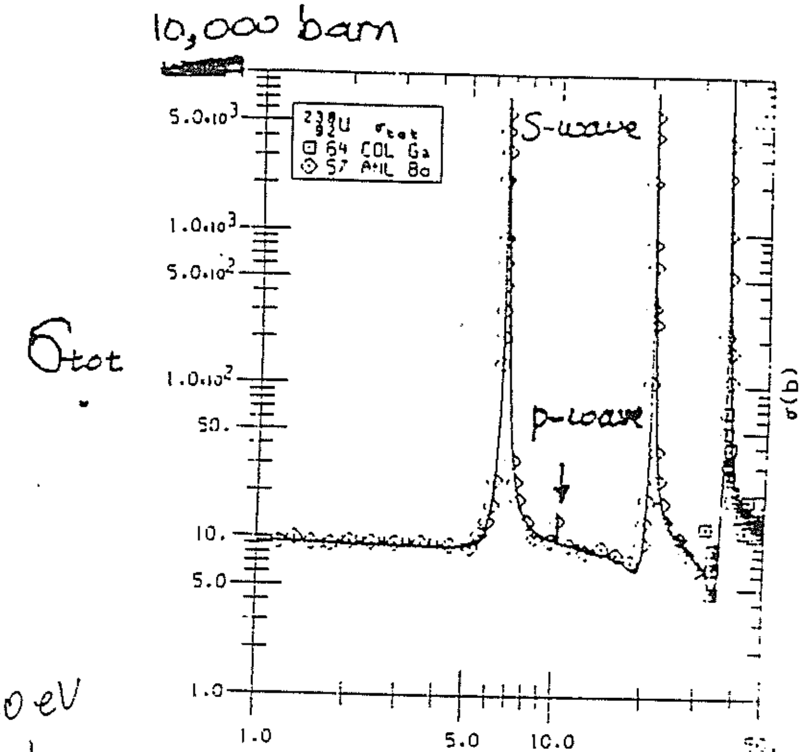
NOW

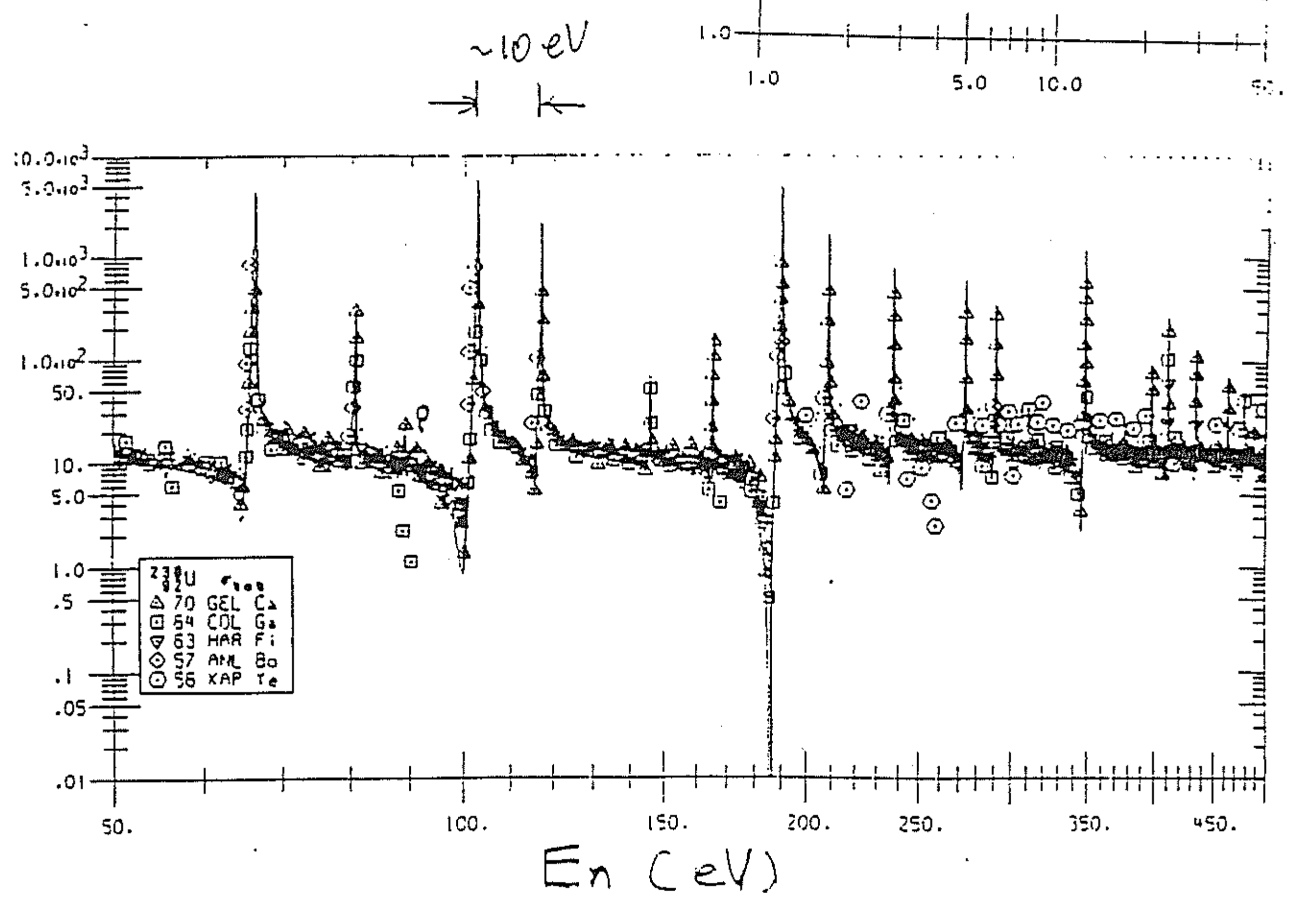
LANSCE

N_TOF (CERN)

- Epithermal (0.1 – 10^5 eV) Neutron–Nucleus scattering:

$n + {}^{238}U$





AFTER MEASURING RESONANCES

HOW TO CATEGORIZE

FIRST ORBITAL ANGULAR MOMENTUM

SIMPLEST

S-WAVES STRONG

P-WAVES WEAK

FORMALIZED WITH BAYESIAN

PROBABILISTIC ARGUMENT
(BOLLINGER- THOMAS)

WEAKEST S-WAVE OR STRONGEST P-WAVE?

VERY DIFFICULT TO GET
SUFFICIENT HIGH QUALTIY P-WAVE DATA

SOLUTIONS

BUT MORE COMPLICATED

PARITY DEPENDENCE

FOR LOW ENERGY NEUTRONS ON HEAVY NUCLEI THE DIFFERENCE IN PENETRABILITY FOR S AND P IS OF ORDER 10,000 OR MORE

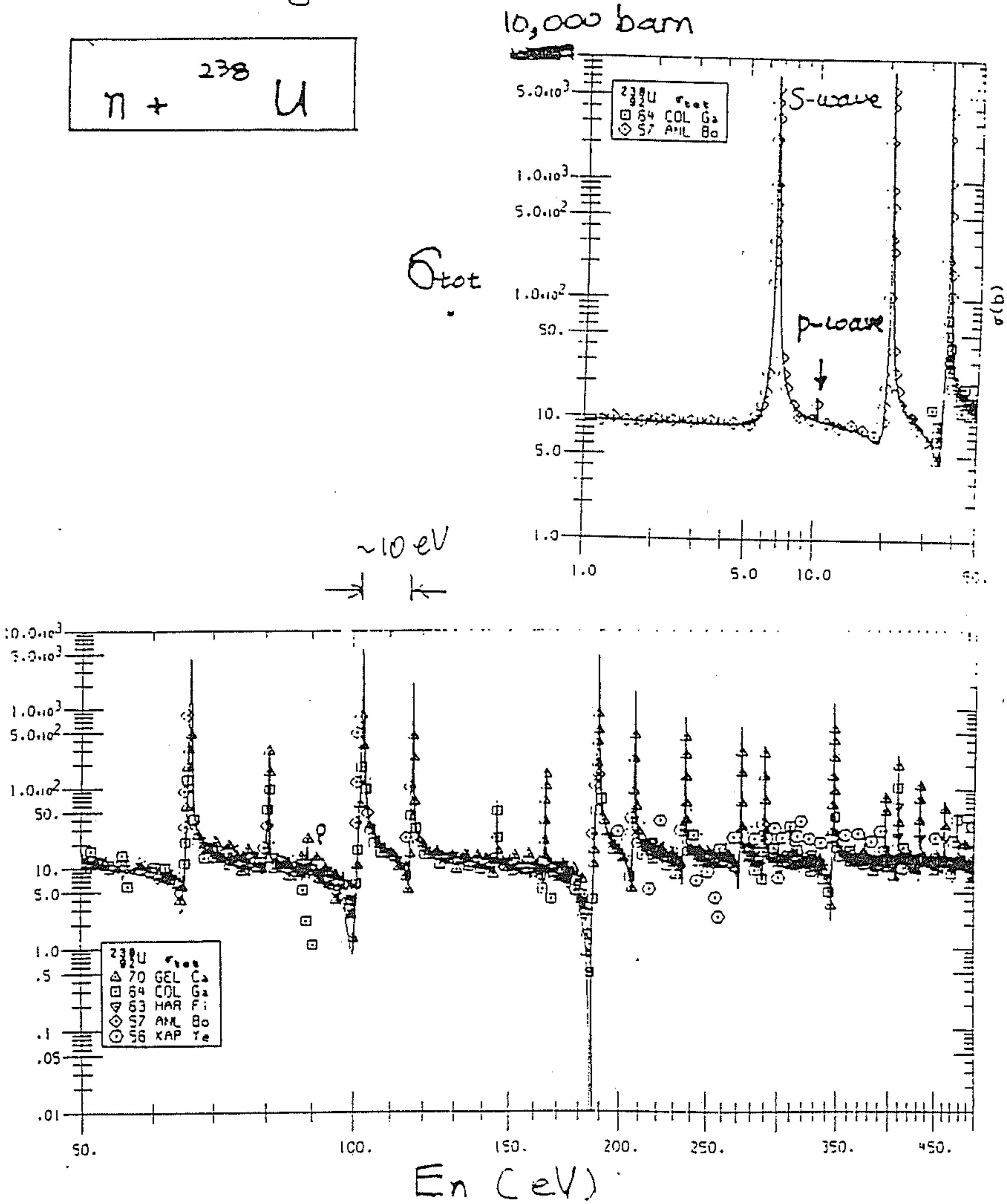
THEREFORE VERY DIFFICULT TO GET GOOD LEVEL DENSITIES FOR DIFFERENT PARITIES

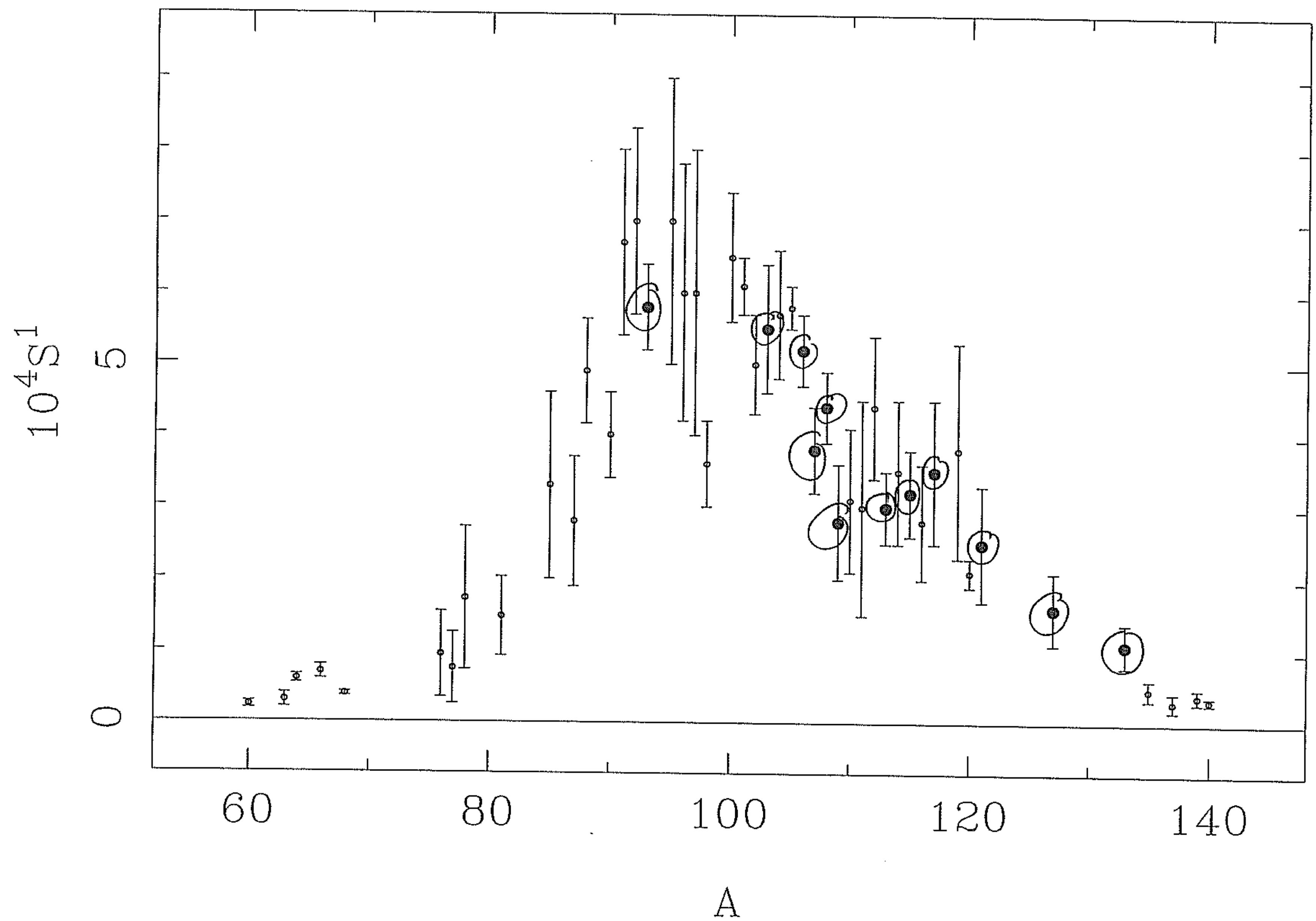
VERY LITTLE P-WAVE DATA
EXCEPT NEAR
STRENGTH FUNCTION MAXIMA

MOST RECENT DATA
BY PRODUCT FROM
"TRIPLE COLLABORATION"
PARITY VIOLATION STUDIES
ON P-WAVE NEUTRON RESONANCES

- Epithermal (0.1 – 10^5 eV) Neutron–Nucleus scattering:

$n + {}^{238}U$





$10^4 S^1$
0
5
60
80
100
120
140
A

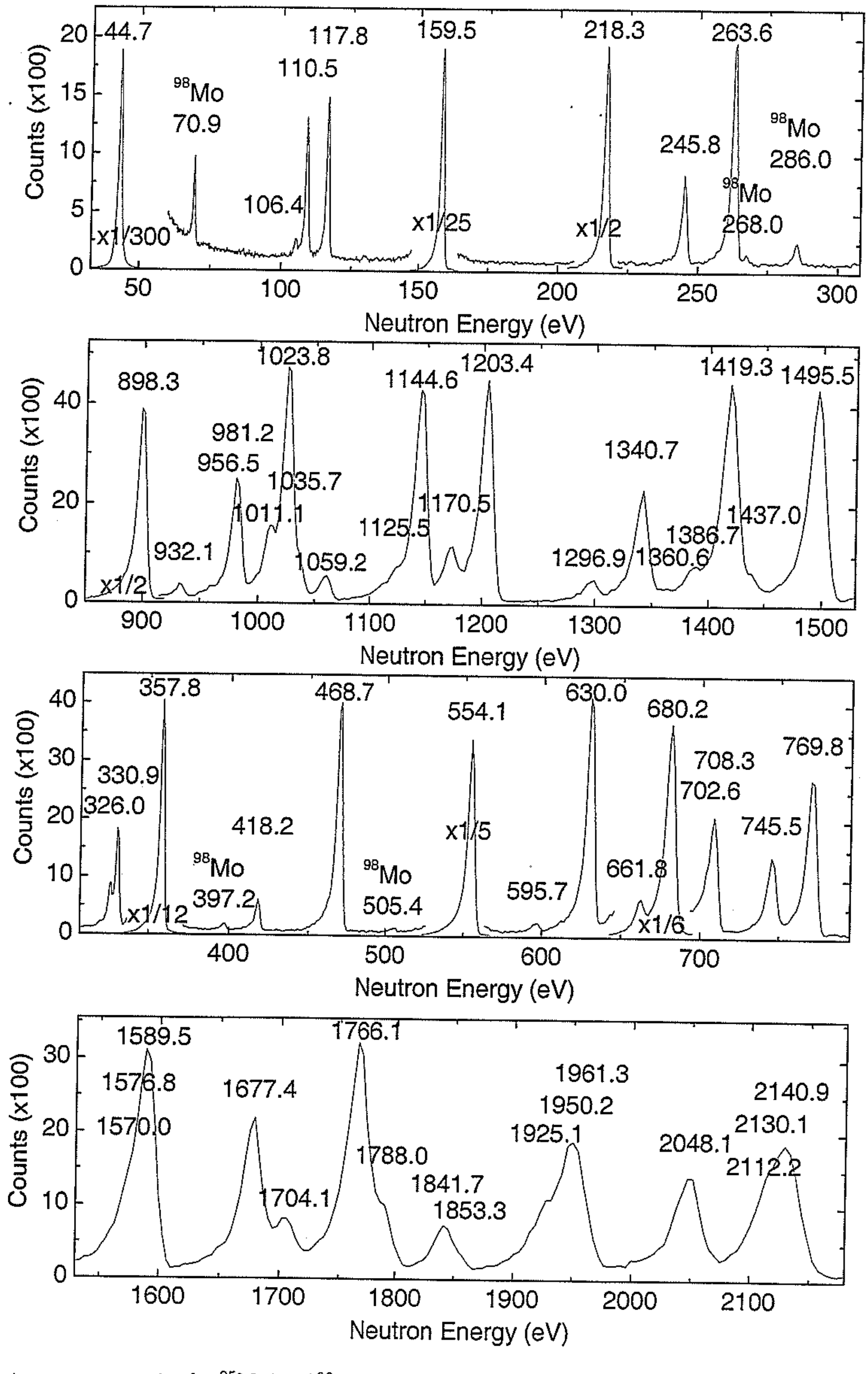


FIG. 2: Neutron resonances in the $^{95}Mo(n,\gamma)^{96}Mo$ reaction: the DANCE detector yield versus neutron energy. The yield is the number of observed capture events per TOF channel with multiplicity $M > 1$ and a γ-ray sum energy from 7.6 to 9.2 MeV.

EXAMPLE 95Mo

GROUND STATE OF 95Mo I = 5/2+

SO J = 2 OR 3

GROUND STATE OF CN 96Mo IS J = 0

ALMOST ALL GAMMA DECAY IS DIPOLE

THEREFORE ONE EXPECTS MORE GAMMA RAYS
FROM DECAY OF RESONANCE WITH J = 3

JARGON -- NUMBERS OF TRANSITIONS TO GROUND
MULTIPLICITY

AVERAGE M VALUABLE BUT NOT SUFFICIENT
WITHOUT KNOWING PARITY

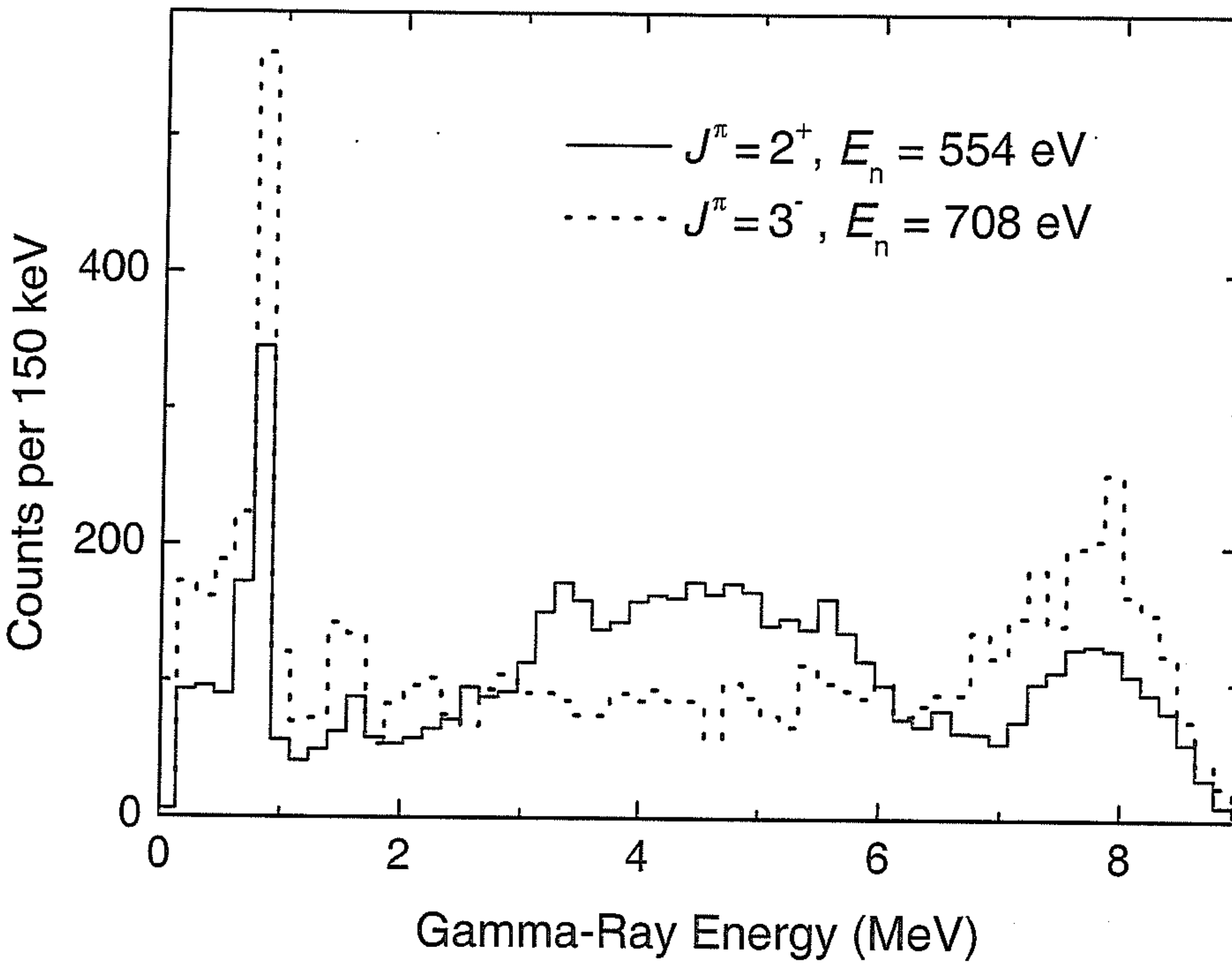


FIG. 5: Measured γ-ray energy spectra for an s-wave resonance at 554 eV ($J^{\pi} = 2^{+}$) and a p-wave resonance at 708 eV ($J^{\pi} = 3^{-}$) in the ^{95}Mo$(n,\gamma)^{96}$Mo reaction. These spectra correspond to two-step cascades with the γ-ray sum energy window from 7.6 to 9.2 MeV.

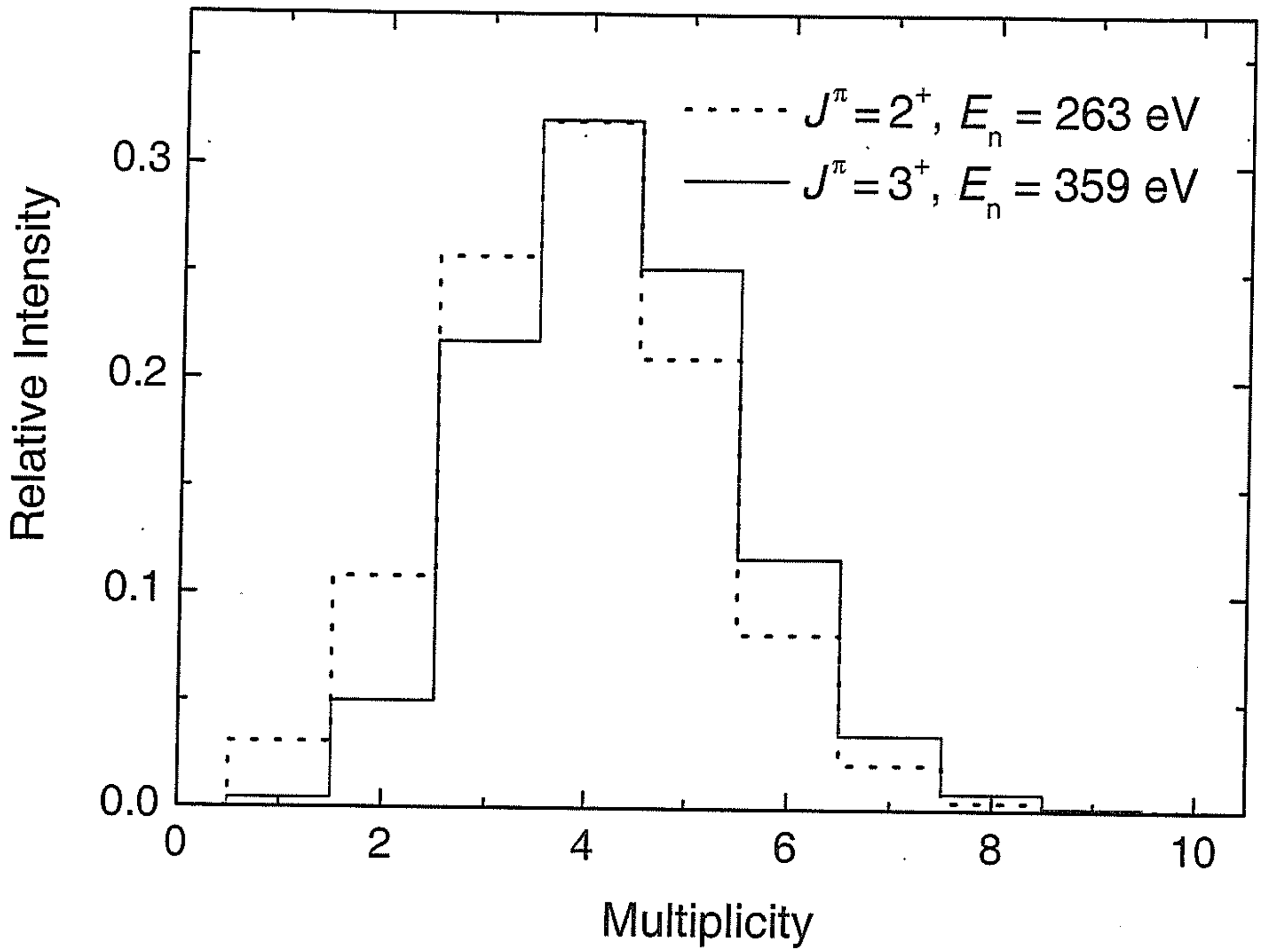


FIG. 3: Multiplicity distributions for s-wave resonances at 263 eV ($J^{\pi} = 2^{+}$) and 359 eV ($J^{\pi} = 3^{+}$) in the $^{95}Mo(n,\gamma)^{96}Mo$ reaction.

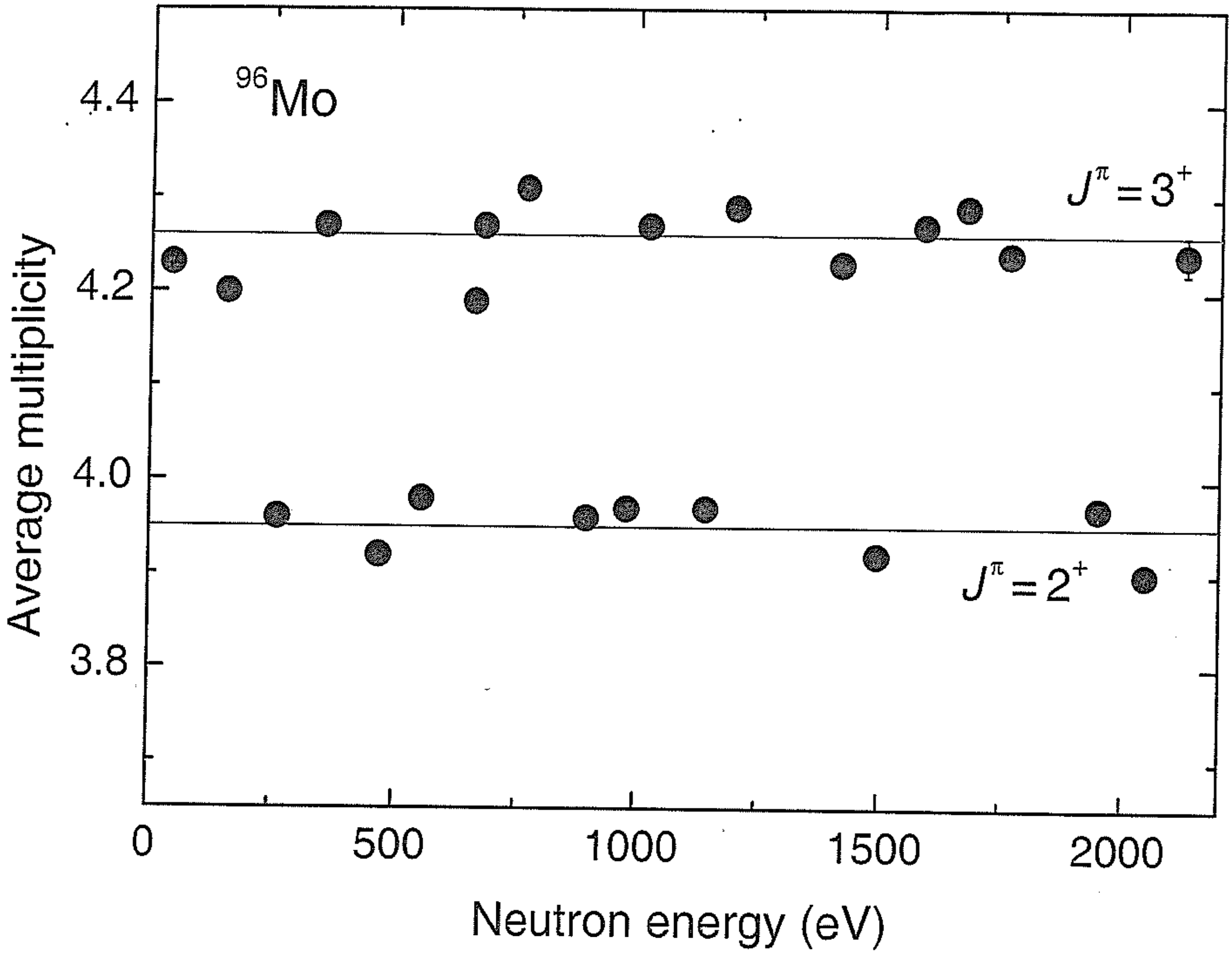


FIG. 4: Average multiplicities of s-wave neutron resonances from the ^{95}Mo$(n,\gamma)^{96}$Mo reaction in the neutron energy range from 40 to 2100 eV. Values along the lower line correspond to spin $J = 2$ and along the upper line to spin $J = 3$.

- Epithermal (0.1 – 10^5 eV) Neutron–Nucleus scattering:

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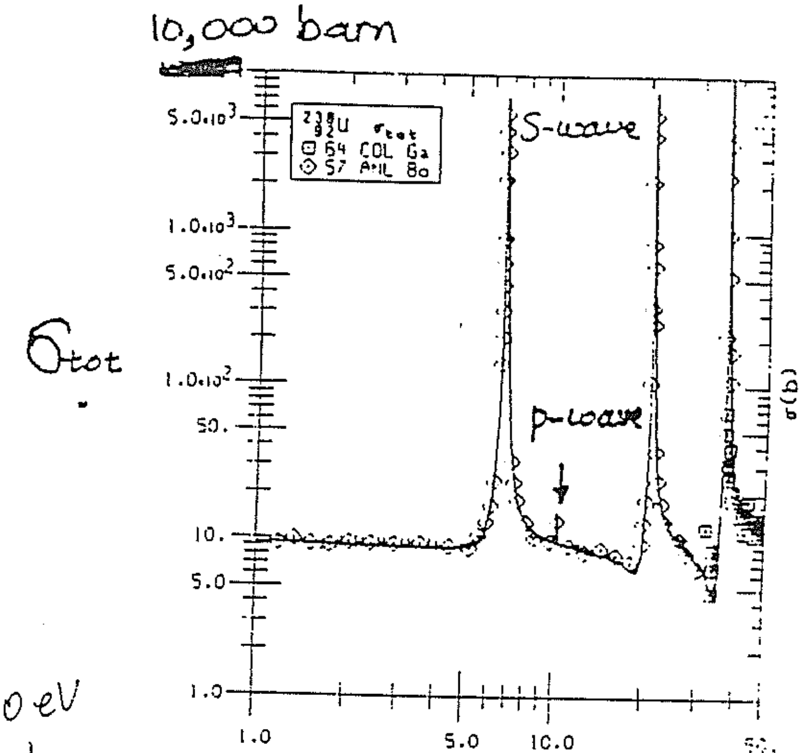
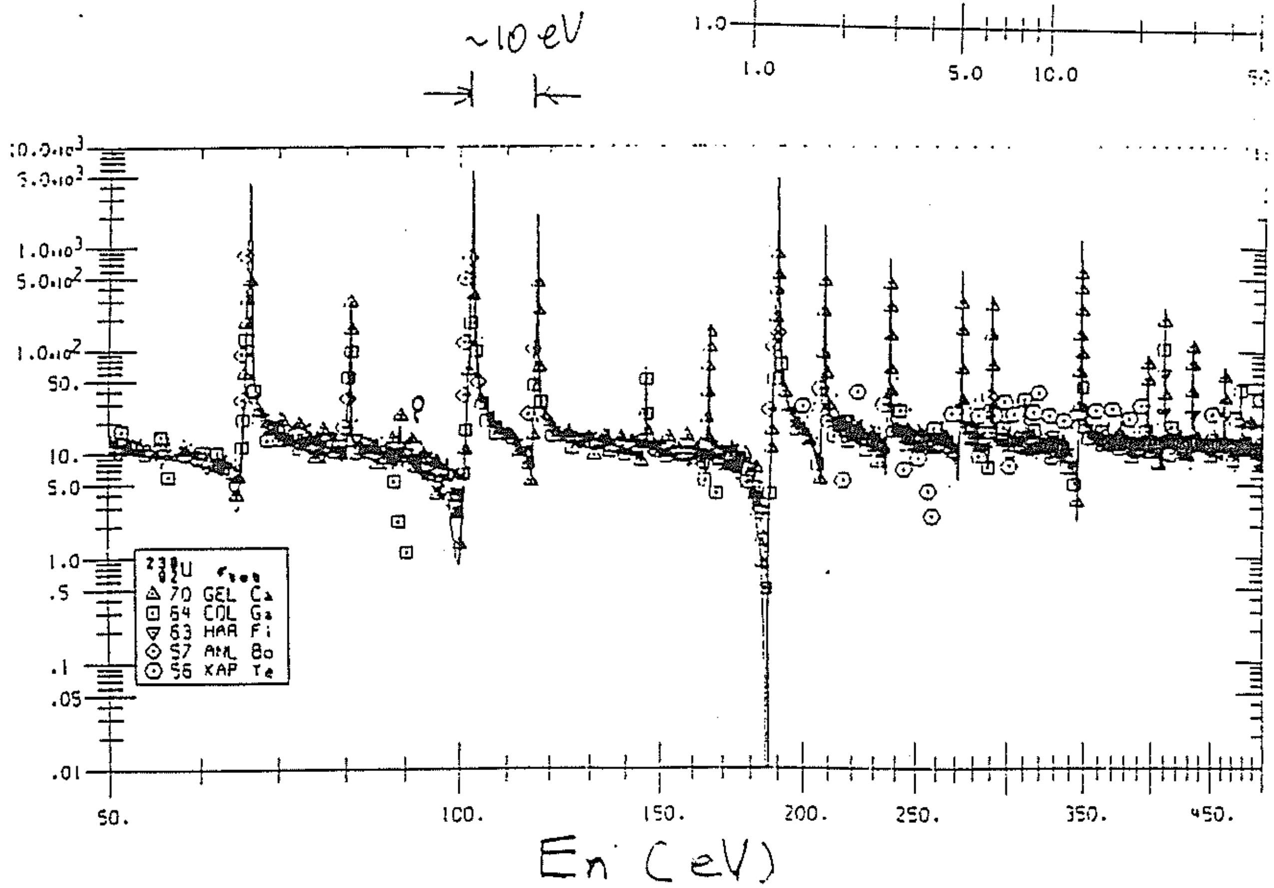




RANDOM MATRICES AND NUCLEI

TWO ROLES

ONE

EVALUATE SPECTRA -- DOES RMT WORK?

IF YES, THEN GENERIC

IF NOT, EXAMINE DYNAMICS

TWO

ASSUME RMT APPLIES

USE RMT TO DETERMINE DATA QUALITY

PROBLEM

RMT APPLIES TO SPECTRA THAT ARE

A -- PURE

B -- COMPLETE

C -- SUFFICIENT STATES
FOR STATISTICAL APPROACH

HOW MANY ARE SUFFICIENT ?
DEPENDS ON MEASURE

PURE - ALL STATES WITH SAME
QUANTUM NUMBERS

COMPLETE – NO MISSING STATES

REAL WORLD

THESE REQUIREMENTS
VERY DIFFICULT TO SATISFY

THEREFORE ISSUE BECOMES

SENSITIVITY OF MEASURES
TO VARIOUS KINDS OF
EXPERIMENTAL LIMITATIONS

TESTS FOR LONG RANGE CORRELATIONS
EXTREMELY SENSITIVE
TO MISSING OR SPURIOUS LEVELS

VERY BAD FOR ROLE 1 –
TESTING SPECTRA

VERY GOOD FOR ROLE 2 –
EVALUATING DATA QUALITY

DOES RANDOM MATRIX THEORY APPLY?

1963 DYSON TESTED COLUMBIA NEUTRON DATA

COULD NOT DECIDE WHETHER RMT APPLIED

LATE 1960S

RAINWATER HAD GOOD NEUTRON RESONANCE DATA
BUT LIMITED STATISTICS

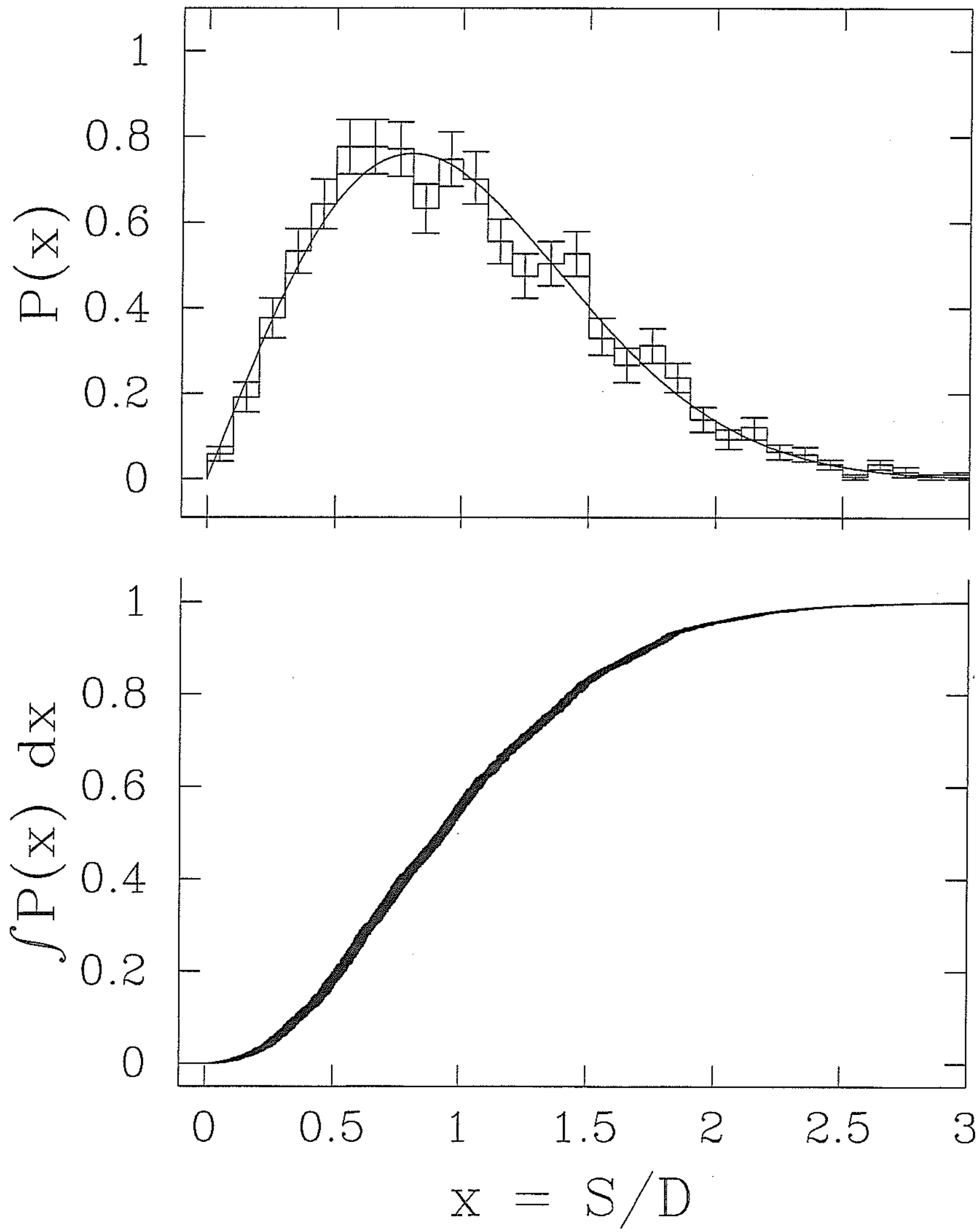
MID 1970s
TUNL HAD GOOD PROTON RESONANCE DATA
BUT EVEN MORE LIMITED STATISTICS

EARLY 1980s

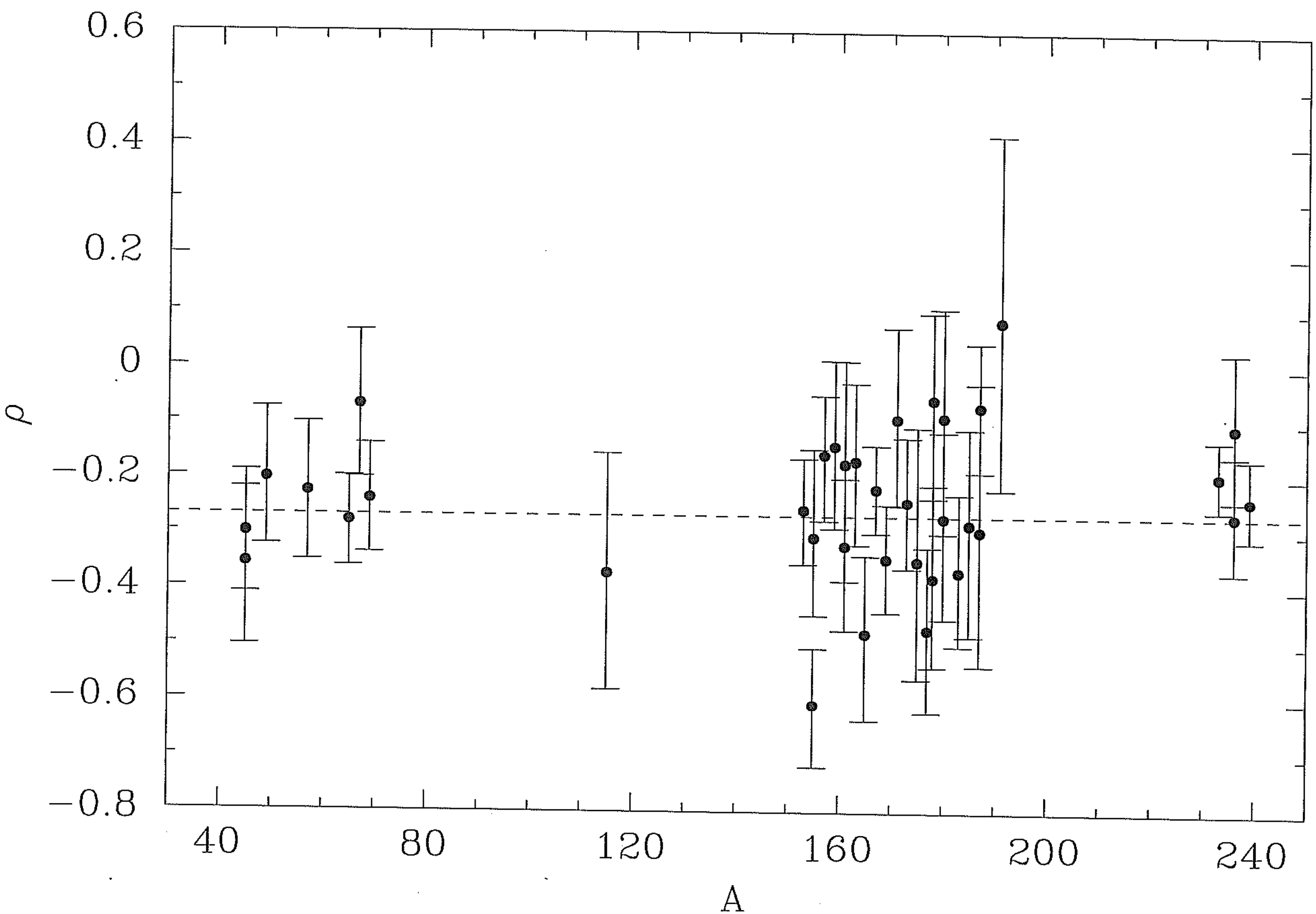
BOHIGAS, HAQ AND PANDEY
COMBINED BEST NEUTRON AND PROTON DATA INTO
NUCLEAR DATA ENSEMBLE

SPACING DISTRIBUTION UNIVERSAL

AGREES VERY WELL WITH GOE

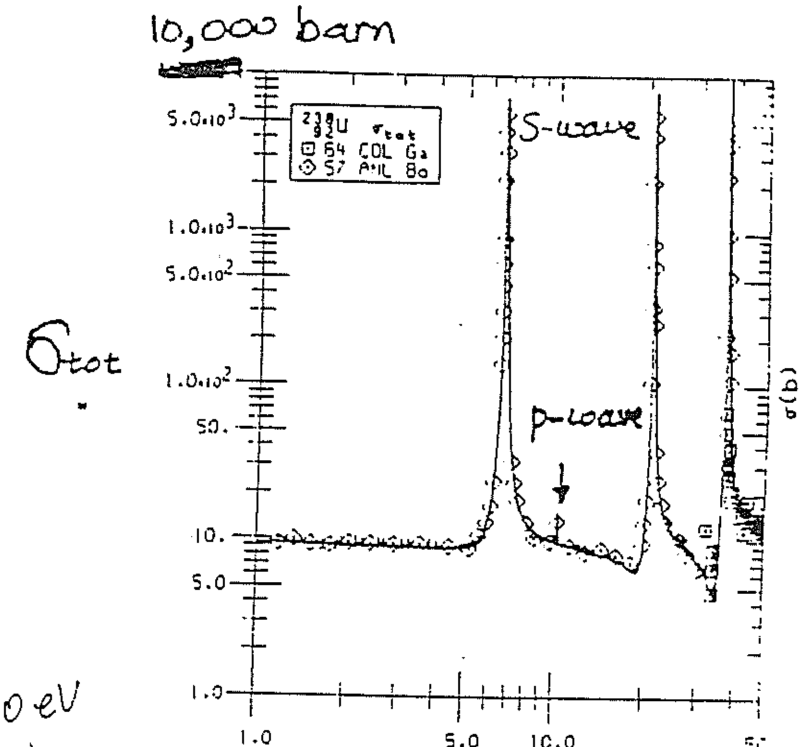


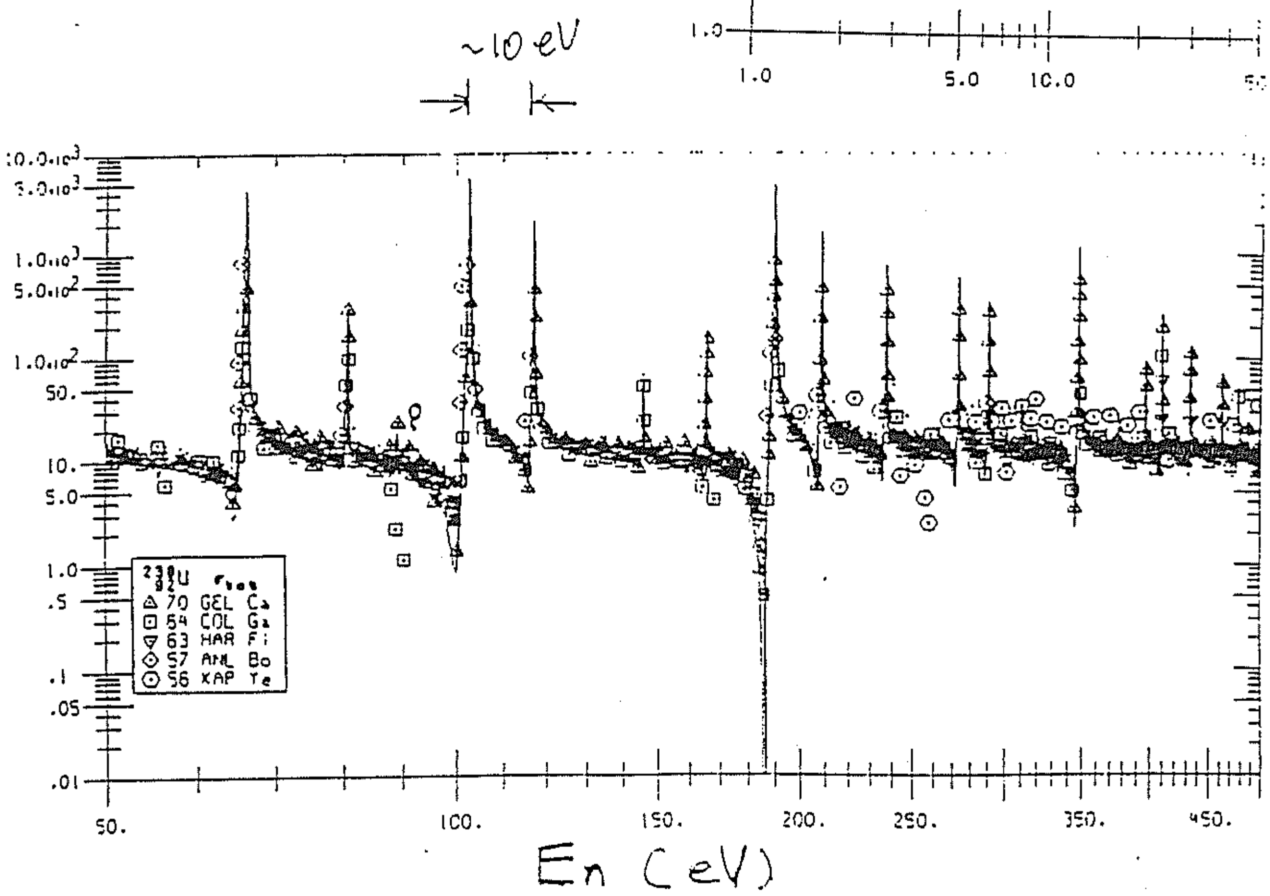
P(x)
∫P(x) dx
x = S/D
0
0.2
0.4
0.6
0.8
1
0.5
1.5
2
2.5
3


0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
ρ
40
80
120
160
200
240
A

- Epithermal ($0.1 - 10^5$ eV) Neutron–Nucleus scattering:

$n + {}^{238}U$





STATISTICAL THEORY OF THE ENERGY LEVELS OF COMPLEX SYSTEMS. V

M. L. MEHTA AND F. J. DYSON
J. MATH. PHYS. 4 (1963)

PROBLEM H. STATISTICAL EFFECTS OF MISSING AND SPURIOUS LEVELS

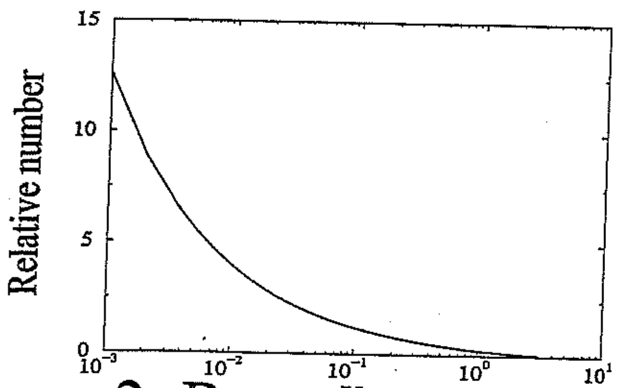
...DESIRABLE TO MAKE THE RESULTS ... MORE PRECISE BY CALCULATING QUANTITATIVELY THE EFFECTS OF MISSING AND SPURIOUS LEVELS. TO CARRY THROUGH SUCH CALCULATIONS WOULD NOT BE DIFFICULT, ONLY RATHER LABORIOUS.

Random Matrix Theory (RMT)

$\Rightarrow$ Assume RMT

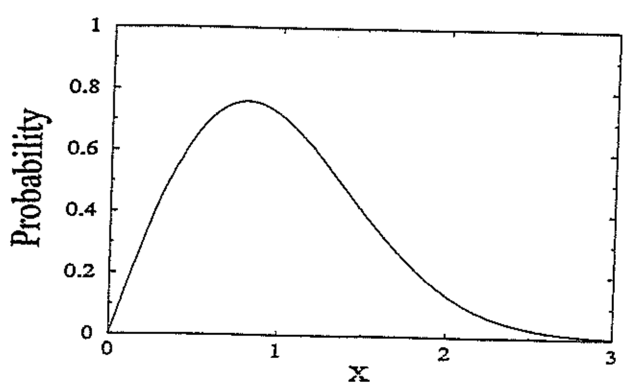
1. Reduced widths obey **Porter-Thomas distribution**

$$P(y)dy = \frac{1}{\sqrt{2\pi y}}\exp(-\frac{y}{2})dy$$



$$y = \frac{\gamma^2}{\langle\gamma^2\rangle}$$

2. Resonance spacings are distributed according to **Wigner distribution**



$$P(x)dx = \frac{\pi x}{2}\exp(-\frac{\pi x^2}{4})dx$$

$$x = \frac{D}{\langle D\rangle}$$

Levels with $y < y_0 = \frac{\gamma_0^2}{\langle \gamma^2 \rangle}$ are undetectable

---

Standard method:

$$\{\gamma_i^2\}, i = 1, N_0 \quad \Rightarrow \quad \langle \gamma^2 \rangle = \frac{\sum \gamma_i^2}{N_0}$$

$$- - - - \longrightarrow \quad y_{\min} = \frac{\gamma_{\min}^2}{\langle \gamma^2 \rangle}$$

$$\textbf{MF} = \text{Missing fraction} = \int_0^{y_{\min}} P(y)dy$$

$$\textbf{MS} = \text{Missing strength} = \int_0^{y_{\min}} yP(y)dy$$

$$\longleftarrow - - - - \quad \langle \gamma^2 \rangle = \frac{\sum \gamma^2 + \text{MS}}{N_0 / (1\text{-}\text{MF})}$$

until converges

USE PORTER-THOMAS
TO DETERMINE MISSING LEVELS

PLUS
ONE MISSES THE WEAKEST LEVELS
USE THAT INFORMATION

MINUS
NONSTATISTICAL EFFECTS
CAN HAVE MAJOR IMPACT

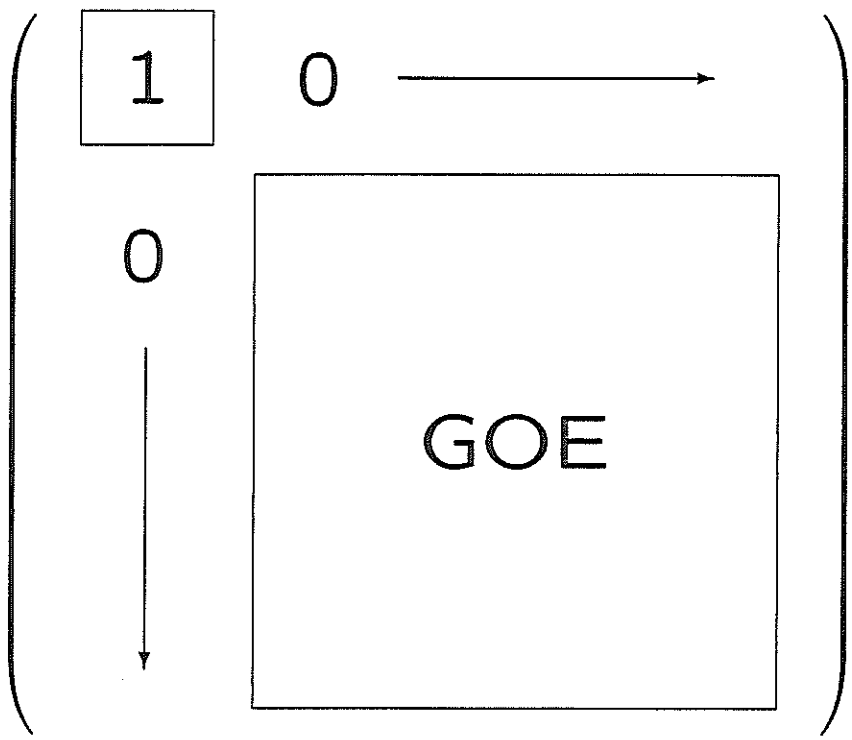
EXAMPLE
DOORWAY STATE
MAY HAVE MANY TIMES
AVERAGE STRENGTH
AND
LARGE EFFECT ON ANALYSIS
FOR MISSING FRACTION

$$\langle \Gamma D \rangle = 0 \Rightarrow \text{use spacing analysis as independent test}$$

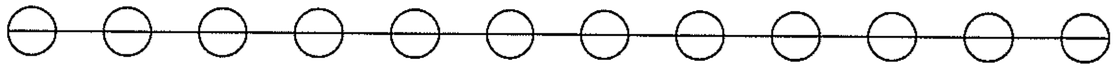
Spacing: *minimal effect due to non-statistical phenomena. However, levels missed at* ***random*** *therefore analysis is harder to formulate*

Question:

Given the spacing distribution and missed levels at random, how can one determine a missing fraction of levels?


1
0
0
GOE

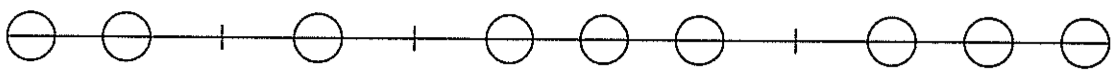
A. Perfect sequence = all levels are observed.



Number of spacings type 0 = 11

$(0+1)11 = 11$

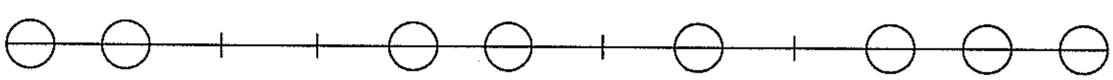
B. Imperfect sequence = levels are missing.



Number of spacings type 0 = 5
type 1 = 3

$(0+1)5+(1+1)3 = 11$

C. Imperfect sequence = levels are missing.



Number of spacings type 0 = 4
type 1 = 2
type 2 = 1

$(0+1)4+(1+1)2+(2+1)1 = 11$

O. Bohigas and M. P. Pato, Physics Letters **B 595**, 171 (2004)

$$\delta_3(L) = (1-f)\,\frac{L}{15} \; + \; f^2\,\Delta_3\!\left(\frac{L}{f}\right)$$

$$\sigma^2(L) = (1-f)\,L \; + \; f^2\,\Sigma^2\!\left(\frac{L}{f}\right)$$

Note linear term

for $f < 1$

Expand the spacing distribution in terms of higher order distributions:

$$P(z)dz = \sum_k a_k \lambda\, p(k, \lambda z)dz$$

Normalizations

$$\int_0^\infty P(z)dz = 1 \qquad \int_0^\infty z\,P(z)dz = 1$$

$$\int_0^\infty p(k,z)dz = 1 \qquad \int_0^\infty z\,p(k,z)dz = k+1$$

lead to conditions:

$$\textbf{(a).} \quad \sum_{k=0}^{\infty} a_k = 1$$

$$\textbf{(b).} \quad \sum_{k=0}^{\infty} a_k (1+k) = \lambda$$

Introduce entropy:

$$S\{a_k\} = -\sum_{k=0}^{\infty} a_k \ln a_k$$

and introduce two Lagrange multipliers α and β (because there are 2 constraint equations)

$$\delta\{S - \alpha \sum a_k - \beta \sum a_k (1+k)/\lambda\} = 0$$

$$\boxed{\begin{array}{l} a_k = (1-f)^k f \\ \lambda = 1/f \end{array}}$$

Using these results and denoting $\lambda z = x$ one obtains:

$$\boxed{\mathrm{P}(x)dx = \sum_k (1-f)^k f\, \mathrm{p}(k,x)dx}$$

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$$S\{a_k\} = -\sum_{k=0}^{\infty} a_k \ln a_k$$

and introduce two Lagrange multipliers α and β (because there are 2 constraint equations)

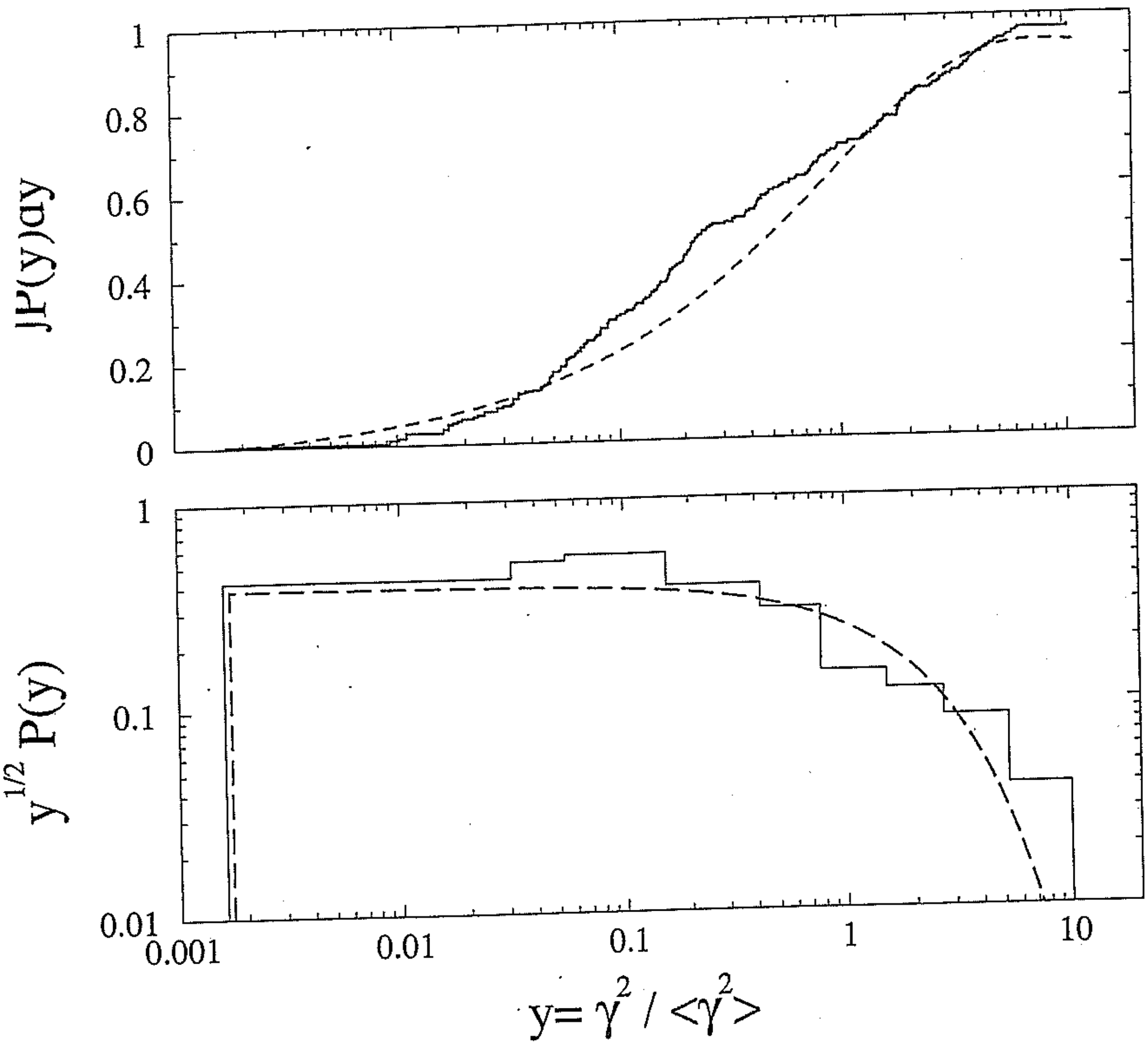
$$\delta\{S - \alpha \sum a_k - \beta \sum a_k (1+k)/\lambda\} = 0$$

$$\boxed{\begin{aligned} a_k &= (1-f)^k f \\ \lambda &= 1/f \end{aligned}}$$

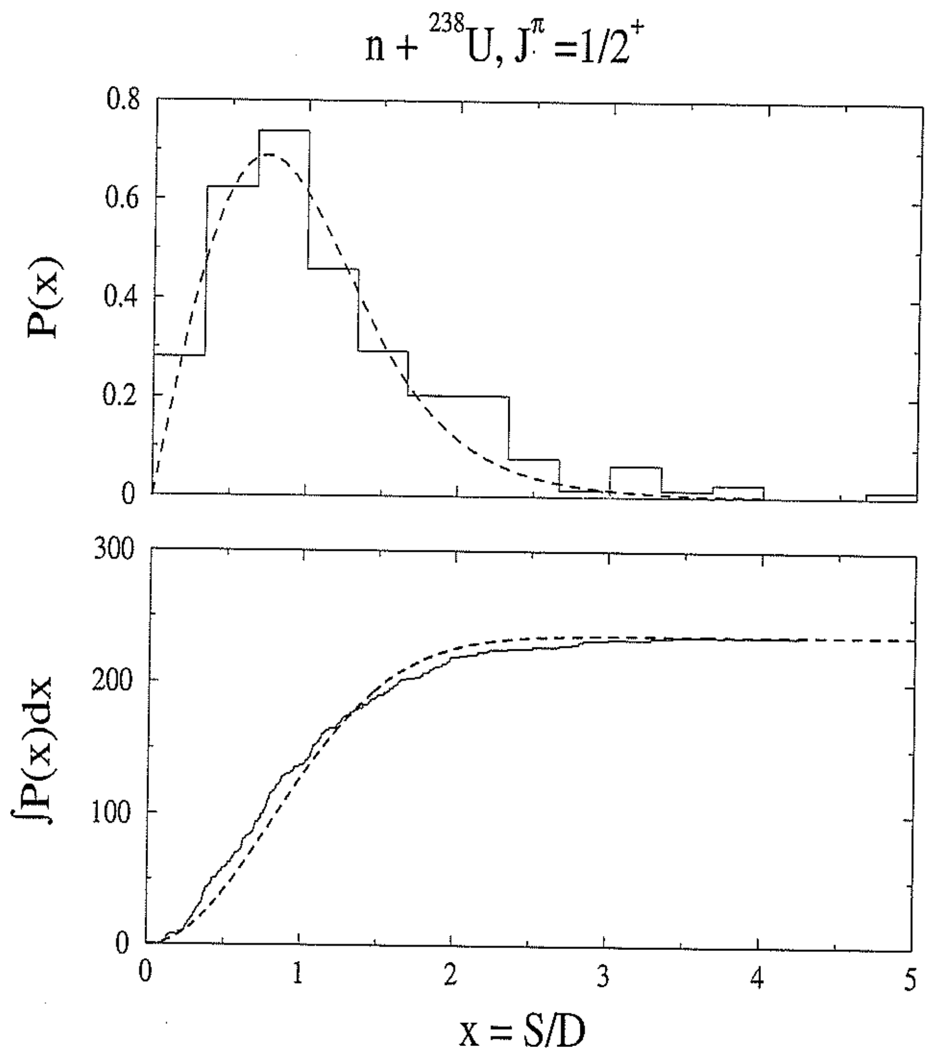
Using these results and denoting $\lambda z = x$ one obtains:

$$\boxed{\mathrm{P}(x)dx = \sum_k (1-f)^k f\, \mathrm{p}(k,x)dx}$$

$n + {}^{238}U, J^{\pi} = 1/2^+$



$f = 0.97^{+0.03}_{-0.08}$



Method 2: Spacing analysis $f = 0.89 \pm 0.06$ less than $f = 0.97^{+0.03}_{-0.08}$ from method 1.