Letter to the Editor

Mathematical Error in "Incompatibility between quantum theory and consciousness" by Daegene Song

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Abstract

In the paper "Incompatibility between quantum theory and consciousness" by Daegene Song is presented mathematical argument according to which quantum theory and self-observing consciousness are incompatible. Here we spot a critical mathematical error in Song's equations, showing that Song's argument is mathematically messed and is quite irrelevant for solving issues concerning consciousness. Moreover, Song's error does not apply specifically for quantum mechanics but for any unitary transformation in configuration space.

Key Words: quantum theory, consciousness, Song's argument

NeuroQuantology 2008; 2:199-201

1. Bra-ket notation

First, let us provide brief description of the mathematical treatment of quantum mechanics provided by Daegene Song in (Song, 2008). For simplicity we shall use bra-ket notation.

Quantum state can be described by $n \times 1$ matrix known as a *ket* collumn vector, e.g.

$$|\psi\rangle = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix}.$$

The *bra* is the *conjugate transpose* of a ket, e.g. $\langle\psi| = \begin{bmatrix} \alpha_1^* & \alpha_2^* & \cdots & \alpha_n^* \end{bmatrix}$.

If $|\psi\rangle$ is an unit vector we have for the inner product $\langle\psi|\psi\rangle = 1$.

Unitary evolution is given by $n \times n$ matrix known as quantum operator U for which $U^\dagger U = I$ is the identity operator. Here $U^\dagger$ is the *conjugate transpose* matrix of U. Quantum operators usually are expressed as outer products of two vectors.

Example: suppose unitary operator U acts on an unit vector $|\psi\rangle$ and transforms is it into $|\phi\rangle$, then $U = |\phi\rangle\langle\psi|$. We can verify immediately that $U|\psi\rangle = |\phi\rangle\langle\psi\|\psi\rangle = |\phi\rangle$. $U^\dagger$ is the inverse operator of U since $U^\dagger U|\psi\rangle = |\psi\rangle$, and it can be directly seen that $U^\dagger = |\psi\rangle\langle\phi|$ that is $U^\dagger$ acts on $|\phi\rangle$ and transforms it into $|\psi\rangle$. Also $|\phi\rangle$ comes to be an unit vector too, and $\langle\phi|\phi\rangle = 1$.

Measurement of state $|\psi\rangle$ in a basis $|\chi\rangle$ gives probability of outcome measured by the squared inner product $|\langle\chi|\phi\rangle|^2$ and the same probability will hold if the inverse is done, namely state $|\chi\rangle$ is measured in a basis $|\psi\rangle$. The probability in this latter case is

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$|\langle\phi|\chi\rangle|^2$, and one immediately sees that $|\langle\phi|\chi\rangle|^2=|\langle\chi|\phi\rangle|^2$.

Now with these introductory notes in mind, let us examine the example **N1** presented by Song.

2. Song's "first natural phenomenon"

Song considers a qubit $|\psi\rangle = a|0\rangle + b|1\rangle$ and operator

$$U=\begin{bmatrix}\cos\frac{\theta}{2} & -\sin\frac{\theta}{2}\\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2}\end{bmatrix}$$

rotating the qubit counterclockwise by angle θ.

He has two vectors $|\psi_1\rangle$ and $|\psi_2\rangle$, such that at time t_0 they are identical $|\psi_1\rangle=|\psi_2\rangle=|k\rangle$.

Then he discusses the rotation applied to the vector $|\psi_1\rangle$ that gives $U|\psi_1\rangle$ at time t_1.

If we want to write that the state vector $|\psi_1\rangle$ is time dependent we can write concisely $|\psi_1(t)\rangle$, which is the shorthand notation for the following:

$$|\psi_1(t_0)\rangle=|k\rangle$$
$$|\psi_1(t_1)\rangle=U|k\rangle$$

And since $|\psi_2\rangle$ is not time dependent we can write $|\psi_2\rangle=\text{const}$, which stands for

$$|\psi_2(t_0)\rangle=|k\rangle$$
$$|\psi_2(t_1)\rangle=|k\rangle$$

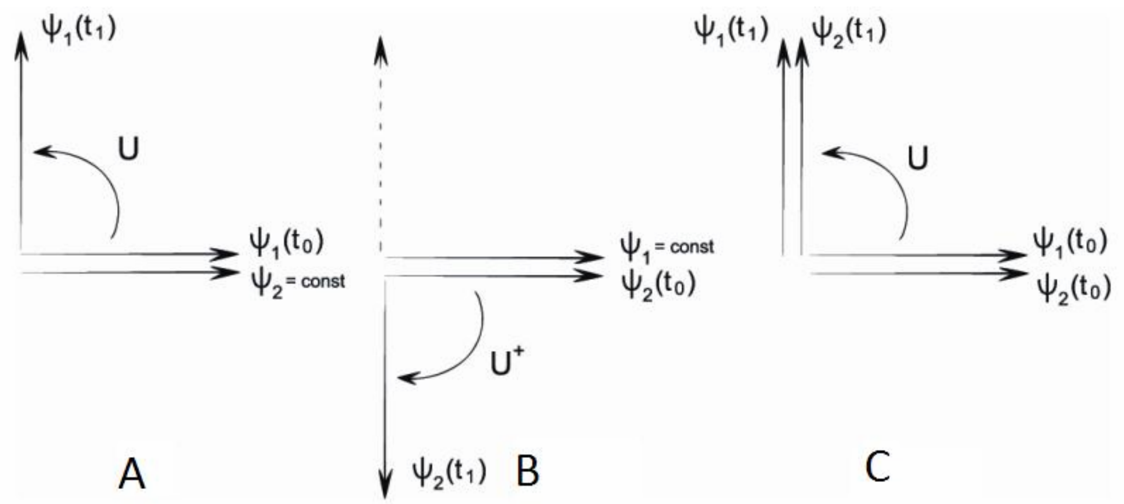
The above clarification is necessary because, as we shall see further, Song does not make difference between time dependent and time independent quantum states.

Now if we measure at t_1 the time evolved state $|\psi_1(t_1)\rangle=U|k\rangle$ onto $|\psi_2\rangle$, we get the probability

$$|\langle\psi_2|\psi_1(t_1)\rangle|^2=|\langle k|U|k\rangle|^2.$$ See fig. 1A.

Next, Song suggests that we should get the same expectation value if the vector $|\psi_2\rangle$ is time dependent but undergoes an opposite rotation $U^{\dagger}$, and then measure the time evolved state $|\psi_2\rangle$ onto the vector $|\psi_1\rangle$ which is considered time-independent $|\psi_1\rangle=\text{const}$. See fig. 1B.

Figure 1. A, B. Song's description of his "*first natural phenomenon*". C. Song's "second natural phenomenon" claimed to be "*self-observation*".



For completeness we can write

$$|\psi_1(t_0)\rangle = |k\rangle$$
$$|\psi_1(t_1)\rangle = |k\rangle$$

that can be shorthand written as time independent state $|\psi_1\rangle = \text{const}$, and

$$|\psi_2(t_0)\rangle = |k\rangle$$
$$|\psi_2(t_1)\rangle = U^{\dagger}|k\rangle$$

that concisely is written as time dependent state $|\psi_2(t)\rangle$. We get the probability

$$|\langle\psi_1|\psi_2(t_1)\rangle|^2 = |\langle k|U^{\dagger}|k\rangle|^2 .$$

3. Song's "self-observing"

While in the previous section we were able to recover what Song wanted to say, in the section **N2** of "self-observation" we see an abuse of mathematical notation. Namely due to postulated *identity* of the basis in which we measure, and the vector that evolves i.e. this is called "second natural phenomenon" or "self-observation", we should write $|\psi_1(t)\rangle \equiv |\psi_2(t)\rangle$. See fig. 1C.

Now it is evident that there are 4 types of observations, or "self-observations".

The "self-observation in the past" that is the past state of the vector, self-observes in the past i.e. strictly we have measurement of the state onto itself, in its own basis.

$$|\langle\psi_1(t_0)\rangle|\psi_1(t_0)\rangle|^2 \equiv |\langle\psi_2(t_0)\rangle|\psi_2(t_0)\rangle|^2 \equiv$$
$$|\langle\psi_1(t_0)\rangle|\psi_2(t_0)\rangle|^2 \equiv |\langle\psi_2(t_0)\rangle|\psi_1(t_0)\rangle|^2 = 1$$

The "self-observation in the future" that is the future state of the vector, self-observes in the future.

$$|\langle\psi_1(t_1)\rangle|\psi_1(t_1)\rangle|^2 \equiv |\langle\psi_2(t_1)\rangle|\psi_2(t_1)\rangle|^2 \equiv$$
$$|\langle\psi_1(t_1)\rangle|\psi_2(t_1)\rangle|^2 \equiv |\langle\psi_2(t_1)\rangle|\psi_1(t_1)\rangle|^2 = 1$$

And there is also a measurement of the past state into the future state basis

$$|\langle\psi_1(t_1)\rangle|\psi_1(t_0)\rangle|^2 \equiv |\langle\psi_2(t_1)\rangle|\psi_2(t_0)\rangle|^2 \equiv$$
$$|\langle\psi_1(t_1)\rangle|\psi_2(t_0)\rangle|^2 \equiv |\langle\psi_2(t_1)\rangle|\psi_1(t_0)\rangle|^2 = \cos^2\theta$$

and inversely a measurement of the future state into the past state basis

$$|\langle\psi_1(t_0)\rangle|\psi_1(t_1)\rangle| \equiv |\langle\psi_2(t_0)\rangle|\psi_2(t_1)\rangle|^2 \equiv$$
$$|\langle\psi_1(t_0)\rangle|\psi_2(t_1)\rangle| \equiv |\langle\psi_2(t_0)\rangle|\psi_1(t_1)\rangle|^2 = \cos^2\theta$$

At this point there is nothing mathematically inconsistent in the axioms of quantum mechanics, and what Song thinks is "self-observation" is not even requiring conscious observer. Due to the postulated identity of the vector undergoing dynamics, and the basis in which the measurement is going to be made, we have to evolve both states with the same operator U, and it is not permissible to use the procedure in the previous section where one applies either U or $U^{\dagger}$. What is more, Song even does not argue that probabilities are different in the **N2** case, but he argues that U and $U^{\dagger}$ transform the vector $|k\rangle$ in two different states. Yes, this is obvious since U and $U^{\dagger}$ are different operators and they rotate the qubit at angles θ and $-\theta$ respectively. But in the **N1** case the same is true. The argument there is that the probabilities for measurement are equal, and not that the counterclockwise rotation is equal to the clockwise one.

As a conclusion, we can say that any sequel of Song's fallacious argument is fallacious too, since it is based on mathematical error.

References

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