Mind Force Theory: roots and developments.

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1. Interacting oscillators.

In an interesting study Ernest Lawrence Rossi (1996) tried to draw the whole picture of the psychobiology of mind-body communication as "the complex, self-organizing field of information transduction". Unfortunately, at the time, he missed most of the contemporary knowledge on synchronization, network theory and field theory, just focusing on information streams and transduction. Anyway, his attempt is interesting, especially for the figure he include which might give us some hint about the scales, clusters and processes we are going to describe as the operations of Mind Force.

Fig. 1 here: heterogeneous couplings in hyperstructure (Rossi, 1996)

It is quite evident that the resulting hypernetwork is formed by different kind of oscillators: relaxation and non-relaxation; chaotic and non-chaotic; with similar and with quite different frequencies etc.

David Somers and Nancy Kopell (1995) studied waves and synchrony in networks of oscillators of relaxation and non-relaxation type. They showed how chains of phase oscillators that phase-lock produce travelling waves. The waves were shown to be a consequence of boundary effects: the oscillators at the ends receive different input than those in the middle, and compensate for the difference by setting up phase differences. They also observed fractured waves and fractured synchrony in networks of relaxation oscillators.

Fig. 2 here: start of synchrony and travelling waves in relaxation oscillators (Somers & Kopell, 1995).

Horacio Zanette has been focusing in this same area of research and in a paper with Alexander Mikhailov they studied condensation in globally coupled populations of chaotic dynamical systems (Zanette & Mikhailov, 1998 a & b). They found that synchronization begins at low coupling intensities with the appearance of small coherent groups of oscillators on the background of the rest of the population performing asynchronous oscillations. The elements belonging to such groups constitute a dynamical condensate. As the coupling strength is increased, the number of particles in the condensate grows and eventually the whole population becomes divided into several coherent clusters. Within each cluster, the elements follow the same chaotic dynamical trajectory. Under further increase of the global coupling, the number of coherent clusters gets reduced until full mutual synchronization is achieved. To quantitatively characterize the condensation, they used two order parameters. The first of them is given by the ratio r of the number of pairs with zero distances to the total number of pairs. In the absence of a condensate, $r = 0$. On the other hand, $r = 1$ when complete mutual synchronization of the whole population takes place. The second parameter s represents the fraction of the population belonging to coherent clusters. It is given by the relative number of elements that have at least one other element with the same state in the considered population. Therefore, s can be viewed as characterizing the size of the condensate.

These forms of synchronization are essential to the functioning of some artificial systems, and have been observed in certain insect populations. On the other hand, they are expected to play a less relevant role in most biological systems, where the complexity of collective functions requires a delicate balance between coherence and diversity. Consider, for instance, the brain, where highly coherent activity patterns are only realized under pathological states, such as during epileptic seizures. Many biological systems consisting of interacting agents, ranging from bio-molecular complexes to social populations, are normally found in configurations where the ensemble is segregated into groups with specific functions (Zanette & Mikhailov 2004).

While the evolution of individual elements is highly correlated inside each group, the collective dynamics of different groups is much more independent. Usually, clustering is a dynamical process, where groups may preserve their identity in spite of the fact that single elements are continuously migrating between them. The individual motion towards or away from clusters may also be controlled by the internal state of each element, which favors or inhibits grouping with other elements. This is observed in natural phenomena ranging from complex chemical reactions, where biomolecules react with each other only when they have reached appropriate internal configurations, to social systems, where the appearance of organizational structures requires compatibility between the individual changing states of the involved agents. The behavior of each single oscillator is usually described in terms of differential equations which can be linear or non linear (the latter usually for biological oscillators). Nonlinear dynamics is the study of the complex ways a system evolves over time. The collective behavior of enormous systems of units has been usually described by statistical mechanics. Network theory is a way to connect these complimentary branches of physics. The analytical techniques of statistical physics can be brought to bear the puzzle of how brain cells and other living things mange to synchronize each other (Strogatz 2003: 55; Albert & Barabasi 2002). Like light in quantum theory, oscillators can be considered as sine waves or as particles. Networks of oscillators could be modeled as spin glasses, or spin foam. We may even recognize how ensembles of oscillators and networks might present peculiar quantum phenomena like the Bose-Einstein condensate, a spontaneous collapse into the same quantum state, the state of lower possible energy (Strogatz 2003: 134). In this state all the waves are locked in step, they are phase coherent.

2. Precursors.

Remarkably little has been written about consciousness in the theory of biological evolution. Richards (1987) captures the core of the problem in his summing up of an argument originally formulated by William James (1879; 1890).

"Consciousness is a manifest trait of higher organisms, most perspicuously of man; like all such traits it must have evolved; yet it could have evolved only if it were naturally selected; but if naturally selected it must have a use; and if it have a use, then it cannot be causally inert. Mind, therefore must be more than an excretion of brain; it must be, at least in some respect an independently effective process that is able to control some central nervous activity" (Richards, 1987: 431). We might add that mind must have a power control over the effectiveness of man on reality.

The idea that an immaterial entity can influence a material entity (reality and the body) is not compatible with an old notion of causality according to which every change in the natural world is produced by contact of spatially extended bodies. This argument was raised by many of Descartes contemporaries. Their primitive antiquated conception of matter as something spatially extended and the related connected notion of causality (as restricted to action by contact) was outpaced by subsequent developments in physics. This conception has not entirely lost its influence in scientific debate and common views. For example P. S. Churchland argues against the existence of "soul stuff" that is not "spatially extended" (1986: 318). Dennett discusses what he calls the *standard objection* which was all too familiar to Descartes (Descartes, 1988; Dennett, 1991) reformulating it in modern terms. In his illustration of this modern criticism, we find the paradox of *Casper the Friendly Ghost* who is both gliding through walls and grabbing falling towels. These contradictory events seem more of a problem for the adherents of mechanistic notions than to those who are familiar with modern physics.

The *analogy* between mind and forces is not an entirely new concept. In philosophy, Hobbes and Leibniz identified a component of mind which they called *conatus,* with a physical force.

3. Fields.

In different ways, some of the key founders of modern psychology, such as Sigmund Freud, William James and Carl Jung, were presenting issues in favor of Mind Force (MF). The history of this construct was progressing during the following years.

During the late 40s, a social psychologist, Kurt Lewin (1890-1947) born in Germany, immigrated to the USA because of World War II. He established also the Research Center for Group Dynamics at

Massachusetts Institute of Technology. Lewin stated that: ‘One should view the present situation—the *status quo*—as being maintained by certain conditions or forces’ (Lewin, 1943a: 172). Lewin, who was well known for using concepts like *life space* and *field theory*, proposed to view the social environment as a dynamic field affecting human consciousness. In turn, the person's psychological state influences the social field or milieu.

Lewin sought to describe group life, and to investigate conditions and forces, which bring about change or resist change in groups. In his field theory, a *field* is defined as ‘the totality of coexisting facts which are conceived of as mutually interdependent’ (Lewin 1951: 240). Lewin believed that in order for change to take place, the total situation has to be taken into account. If only part of the situation is considered, a misrepresentation of the picture is likely to develop. The whole psychological field, or *lifespace*, within which people acted, had to be viewed, in order to understand human behavior. Within this framework, individuals and groups could be seen in topological terms (using map-like representations). Individuals participate in a series of life spaces, (such as the family, work, school and church) and these spaces were constructed under the influence of various force vectors (Lewin 1952). His active approach could be summarized in his motto: “Learning is more effective when it is an active rather than a passive process: if you want to truly understand something, try to change it” (Lewin, 1952).

Harry S. Sullivan (1892 – 1949), like Lewin, was concerned about using theoretical constructs featuring falsifiable reference in interpersonal behavior. While Sullivan may not have read Lewin, Wertheimer, Kohler, or Koffka, he almost certainly would have had welcomed their general approach, since he was proposing a psychological field theory similar to their psycho-physical field theory. He may have seriously intended only a loose heuristic function in casting interpersonal relations as occurring in a “field.” Nevertheless, he was intrigued by this notion. It seemed to him that the field provided an accurate and useful description of effects. Though he didn’t write much about fields and mind force, during his brief lifespan, he left a wonderful sketch presented during one of his last lectures (LaForge, 2004).

Fig. 3 here: H. S. Sullivan sketch on mind force fields in a couple (LaForge, 2004)

4. Force

More recently, the philosopher Karl Popper has emphasized the similarities between mind and forces: “Minds are located, unextended, incorporeal, capable of acting on bodies, dependent on body and capable of being influenced by bodies. (…) Now, I say, things of this kind *do* exist, and we all know it. So, what are these things? These things are forces” (Popper, 1993: 168).

Popper seems to go further than a mere analogy, and he proposes, as a hypothesis “that the complicated electro-magnetic wave fields which, as we know, are part of the physiology of our brains, represent the unconscious part of our minds, and that the conscious mind – our conscious mental intensities, our conscious experiences – are capable of interacting with this unconscious physical force fields, especially when problems *need* to be solved. That *need* is what we call *attention*" (Popper, 1993: 179).

Fig. 4 here, Karl Popper's scheme

Here Popper is considering unconscious as synonymous with the physical force fields. In the figure in the following page, you might notice that there is an area of mind/brain overlap. This area represents a form of biophysical unconscious. Popper points out that conscious mind may "sink into physiology" and become unconscious: "a mergent process, a process where (unconscious) mind and brain are no longer distinguishable" (Popper 1993: 171).

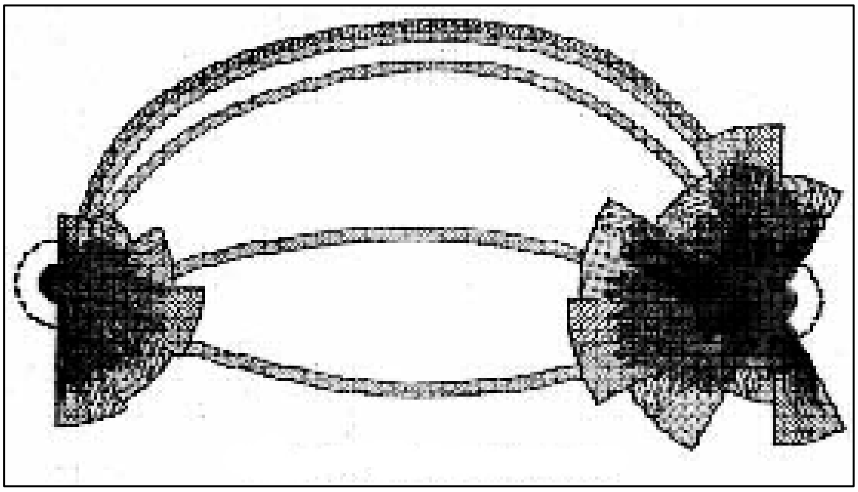

Fig. 3 H. S. Sullivan sketch on mind force fields in a couple (LaForge, 2004)

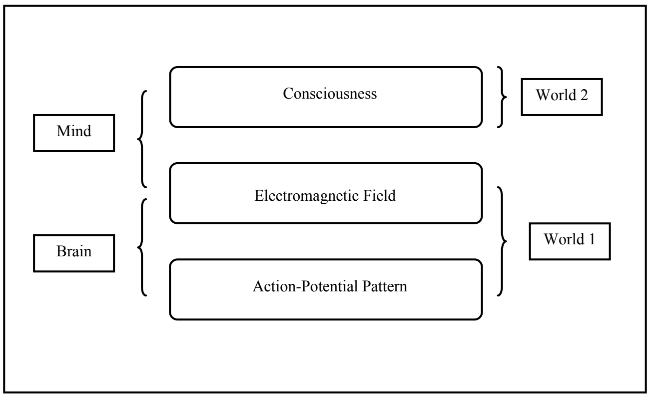

Fig. 4, Karl Popper's scheme

In explaining part of his conception of unconscious, Popper utilized the example of learning a complex psychomotor skill (playing the piano, grasping a mug, and a lot of other activities). In the first stages of the learning process we are conscious and our attention is focused on each single step in the new skills to learn. This stage, sooner or later, disappears and we no longer think about each single step. The new acquired skills are now embodied and, we might say, merged in our being. This process is part of the constitution of *procedural memory*.

There is no need to postulate a mind force in order to explain procedural memory, so Popper's example might be misleading. His proposal of founding mind force on electromagnetic interactions is also improbable for physical reasons. Anyway, some of Popper remarks and his endorsement of the necessity of MF theory are certainly important.

4. Conscious Mental Fields

Benjamin Libet, in the same years, had also proposed the hypothetical existence of a *Conscious Mental Field* (CMF) (Libet 1993, 1994). The CMF would emerge as a function of neural activities in the brain and it would have the attribute of a conscious subjective experience. Libet also suggested that it could act back on certain neural activities and would therefore affect behavior, as in a willed action. It would account for the unity of a subjective experience, even though the latter emerges from the myriad of activities of billions of nerve cells and their synaptic and non-synaptic interplays.

In Libet's opinion "The CMF, like the subjective experiences constituted in it, would be accessible only to the individual having the experiences it could not be directly observed by any external physical device except indirectly, by any effects it introduces on behavioral outcomes (just as conscious will is evidenced)" (Libet, 1996: 223).

In a paper entitled "Mind as a force field: a new interactionistic hypothesis", B. Lindahl and P. Arhem (1994) proposed a critical review of some modern approaches to Mind Force. Libet in his response to their remarks (1996) wrote that he liked "Popper's idea of viewing the mind as a kind of force field". Such a CMF force would have been different from all known physical forces, though Popper's hypothesis does not appear to spell out any attributes of that conscious physical force field except in its ability to interact with another entity. The electromagnetic field representing the unconscious mental functions is "doubtful based on evidence available". Consciousness, following Libet's experiments, can simply be a function of the duration of cerebral activations to achieve awareness. Libet was stressing that, "Evidence suggests that conscious functions involve some special neural activities that are simply added to those involved in conscious functions" (1996: 224). Libet is posing two important questions: first, how does the CMF arise out of mental activities; and second, exactly how the CMF does act on the physical brain.

Nevertheless, it seems that Libet was forgetting the dynamical nature of CMF and the self-organizing nature of its power. He does revisit the issue, reminding, "whether electromagnetic fields are representative of unconscious mental functions could be tested in principle by experimentally distorting and/or disrupting or modifying such fields in the putative relation to unconscious functions". Sperry (1947) had already tried such experiment by cutting the monkey cortex in slices. However, Sperry's vertical cuts in the cortex may not have affected larger field currents as electrical pathways over and below the cuts were still present as a potential role for over-arching electric fields therefore remains possible.

5. The Aconscious

Lindahl & Arhem (1996) noted that "in the mental force field hypothesis only the unconscious part of mind is explicitly interpreted in field terms". They correctly point out that Libet (1996) mistakenly assumed that the Popper force field hypothesis applied to the conscious mind. Popper left open the question of the nature of conscious mind However; he did except to say that the mind in its general form emerges from the body, somehow, but is not reducible to it. What Libet misunderstood, and it might be not clear enough in Lindahl & Arhem response, is that the unconscious in question should be properly named *aconscious.* Its contents and processes are not repressed, and possibly recalled to consciousness, but rather embedded in the human psychomotor structure via psycho-physiological storage.

Libet, more recently (2006), has summarized his view on Mind Force. He starts recognizing that, if there is a generally held assumption that mind and brain can interact, this indicates that "two phenomenological entities exist". After this "back to Descartes" statement, he continues with the questionable assumption that mind, in his view, is just a subjective experience, accessible only by individual introspection.

This point of view, discards all the psychological and linguistic research studying psychomotor, perceptual and linguistic manifestations or derivatives of mind functioning. His point recalls the "private language" paradox proposed by Ludwig Wittgenstein (1967). The conclusion raised by this paradox is that every form of language and thinking (even the so-called inner dialogue) relies on our shared forms of communication. Language (also in the form of culture) is pre-existing to the subject, though its communicative or private usage for thinking can be personalized. If the idea of a private language is incoherent, then it would follow that all language is essentially public: that language is at its core a social phenomenon. For instance, if one cannot have a private language, it might not make any sense to talk of private sensations such as *qualia*; nor might it make sense to talk of a word as referring to a concept, if a concept is understood to be a private mental representation.

Libet recalls the interest that Sir John Eccles derived from Sir Charles Sherrington for the interactions between brain and mind. When the physicist, Henry Margenau, provided a view of the mind as a field that could interact with the brain even with no energy expenditure (Margenau, 1984), this supported Eccles' bias on the nature of mind–brain interaction.

Libet notes, “It is especially noteworthy that Eccles’ models of mind–brain interaction were presented without any experimental evidence or experimental designs for testing. That was due at least partly to the untestability of the models. Curiously, an absence of experimental testability did not bother Eccles. When asked if his view that a field of ‘‘psychons’’ (his units of mental function; see Wiesendanger, 2006) could mediate the unity of subjective experience (Eccles, 1990) was untestable, Eccles replied that he knew of no way to test that hypothesis (personal communication). Nevertheless, he argued that the hypothesis had explanatory power, and, as such, he believed it had some usefulness and even validity. Eccles produced a stimulus further contribution to work in the direction of an MF definition, but his models remained untested, and, apparently, he was not bothered about it.

Libet raises the testability issue also about the approach proposed by Hiromi Umezawa and his followers: as they proposed a mental field model, which they termed a *Quantum Field Theory* (claimed by this group of authors as different from *Quantum Mechanics*) (Ricciardi & Umezawa, 1967; Umezawa & Vitiello, 1986; Vitiello, 1995; 2001; 2002). In Libet’s opinion, “Their model is mostly mathematical, however, and it is not clear how it can be tested” (Libet, 1993; Libet, Freeman & Sutherland, 1999). Libet is raising a similar objection against the quantum mechanics approach as in the interpretation of quantum theory by Nils Bohr (1885–1962) mind and matter are two aspects of one undivided process. David Böhm (1917–1992) adopted this idea (see Böhm and Factor, 1985). However, this does not solve the problem of how the neuronal activity aspect relates to the subjective, non-physical aspect of mind. If subjective experience is a non-physical phenomenon, what is it? The merit of the Bohr-Böhm approach is in recognizing that there is a physical process behind and beyond mind and brain.

Libet claims that his CMF theory is potentially testable as he described a design for conducting such tests. The proposed experimental test is simple in principle but difficult to carry out, a small slab of sensory cortex, which keeps the tiny cortex island alive by preserving the blood vessels providing blood flow from the arterial branches that dip vertically into the cortex. “The prediction is that electrical stimulation of the sensory slab will produce a subjective response reportable by the subject. That is, activity in the isolated slab can contribute by producing its own portion of the CMF.” (Libet, 2006: 324). He states, and we agree with him, that his CMF is an emergent and localizable system property. Less clear is why this experiment, which doesn’t seem technically so difficult, has not been performed yet by other scientists or Libet himself.

Libet is also referring to the functioning of the CMF as the delay in sensory awareness of 0.5 s after the initial response of the cortex as well as the other very interesting phenomenon related to readiness for action, which is preceding actual actions by about 300 ms. So, both perceptual and motor activities have significant delays with consciousness. *These empirical findings support the autonomy of aconscious Mind Force processes from consciousness processes.* This landmark, (though still partially controversial) findings by Libet, could entirely re-design the role of conscious and unconscious processes. The term unconscious here can be mistakenly confused with the traditional (repressed) unconscious, but in this case, we are dealing with events without any conscious representation. Indeed, most mental events are unconscious, or we might better say, *aconscious* (Orsucci, 2002b; Orsucci, 2002a). Libet concludes his review mentioning in vivo and brain imaging research supporting his findings. "If an experimental test of the CMF was to be carried out, like that described above, it might confirm or contradict the kind of alternatives possible for a mind–brain interaction" (Libet, 2006: 326).

6. Energy

Walter J Freeman, after his explorations on mass action in the nervous system, chaos dynamics in perception and social dynamics, has more recently proposed a Mind Force approach (Freeman, 2007). He first defines the framework of this approach: "Consciousness fully supervenes when the 1.5 kgm mass of protoplasm in the head directs the body into material and social environments and engages in reciprocity. While consciousness is not susceptible to direct measurement, a limited form exercised in animals and pre-lingual children can be measured indirectly with biological assays of arousal, intention and attention." It is a remarkably non-deterministic and interactionistic approach.

After a general description of the multiple levels of interactions involved, from the molecular to the social levels, including their intermingling, he states: "Every reflex and intentional act and thought is based on the exchanges of matter and energy through neural activity at every scale." (Freeman, 2007: 1022). There is a need for a universal language to comprehend all the incredible mesh of interactions involved and the mathematical tools needed might be already at hand,.

There is no definition of what consciousness is and, no physiological or cognitive index of consciousness, as many discussions on consciousness still tend to confuse it with self-consciousness. So, Freeman states that he wants to consider the perceptual and behavioral derivates of consciousness that we might find even in infants and animals, "I leave the hard problem (Chalmers, 1995) to philosophers."

He stresses that the stream of consciousness is cinematographic, as we have seen in chapter 2 rather than continuous. Consciousness role in human behavior is judgmental rather than enactive, so that its prime role is not to make decisions but to delay and defer action and thereby minimize premature commitment of limited resources. Just as we use to say in the adage, "stop and think before acting". Following this path, Freeman comes to a clear statement: "consciousness is not merely 'like' a force; it is a field of force that can be understood in the same ways that we understand all other fields of force (Freeman, 2004) within which we, through our bodies, are immersed, and which we, through our bodies, comprehend in accordance with the known laws of physics." (Freeman, 2007: 1022).

The models that Freeman has implemented are schematized in two ways: one is the so called Katchalsky model (or K-sets) (Freeman, 1975; Kozma & Freeman, 2003); the other one is the quantum field model he more recently developed in collaboration with Giuseppe Vitiello (2006).

In Freeman's description Mind Force might be found in the action–perception cycle as described by Piaget (1930) and Merleau-Ponty (1942). The cycle begins with a macroscopic state in the brain that embodies a goal. It emerges in the brain from a predictive state implicitly containing nested mesoscopic activity patterns, constructed in corticostriatal and corticocerebellar modules (Houk, 2005). The predictive expectations embedded in sensory cortices are described as landscapes of chaotic attractors within the brain state space. The dynamic memory embodied in nerve cell assemblies is manifested in spatial pattern of amplitude modulation, mostly in the gamma band range. An interesting property of the system is that these dynamical landscapes lack invariance, as they change whether the same stimulus is reinforced or not, or the context is different or the sequence of stimuli is different.

"The attractor governs the neural interactions that generate an oscillatory field of neural activity called a wave packet" (Freeman, 1975). Fields are not fixed representations of the stimuli, and stimuli are not grounded in any fixed way. Each action-perception frame is separated from the others by phase transitions. Freeman cites Wolfgang Köhler who was (1940: 55) quite explicit about this: "Our present knowledge of human perception leaves no doubt as to the general form of any theory which is to do justice to such knowledge: *a theory of perception must be a field theory*. By this we mean that the neural functions and processes with which the perceptual facts are associated in each case are located in a continuous medium". Regrettably, Köhler identified his perceptual field with the epiphenomenal electric field of the EEG, of which the Coulomb forces are much too weak to synchronize the observed oscillations in wave packets (Freeman & Baird, 1989). Sperry (1980) and Pribram (1971) easily disproved this subsidiary hypothesis, with the unfortunate outcome that mainstream neuroscientists largely abandoned field hypotheses.

7. Four Pillars.

Complexity theory has provided the empirical and mathematical tools to prove that the brain patterns correlated to the cine-like frames in the action-perception cycle are like bubbles in a pan of boiling water at the critical temperature. They can be seen also as the avalanches on a sand pile, as in the model of *self-organized criticality* proposed by Per Bak (Bak et al., 1978; Bak, 1990; Bak, 1996).

From this neurodynamical point of view, the personal identity is "embodied in the entirety of the brain-body dynamics" this is the reason why we have been speaking about a comprehensive *biophysical identity*. We find this definition more grounded and explicit, but in the same line of the approach suggested, with different nuances, as *embodiment* (Varela, Thompson, & Rosch, 1991), *proto-self* (Damasio, 1999), *global workspace* (Baars, 1999), *synaptic self* (LeDoux, 2002). Following the history of the Mind Force construct we might realize how it come comes gradually to take the shape of a new theory, approaching the stage in which it can be formalized. We can summarize some of the necessary prerequisites of this theory.

In order to recognize the existence and operability of MF and its related phenomena, we need to accept and stabilize a definitive transcending of the notorious Cartesian dichotomy. The current discussion on

Descartes' error is often missing two important points: the heuristic value that his position has had for centuries in the advancement of science; and a full recognition of all the implications that discarding his approach will have on our new scientific approaches. MF is beyond *Res Cogitans* and *Res Extensa* in dynamical and structural terms: we say that it constitutes a superior dynamical unity.

New physics and new biomedicine gave us some crucial tools to transcend Descartes. The immense complexity and dimensionality of human systems, if considered in a post-Cartesian view, must be studied in the modern terms of complexity theory, nonlinear dynamics, field theory, quantum mechanics, molecular biology and cognitive science. These seemingly different approaches are integrated in order to reach a real view of MF nature and operations, beyond the dichotomies and appearances we are used to see.

A logical consequence is that MF, in its structure (that we recognize in networks) and dynamics (that we recognize in fields / waves of synchronizing nonlinear oscillators), would be heterogeneous. Dynamics, fields and the hyperstructure of MF would span through molecular domains, neural domains, cognitive domains and even socio-cultural domains. We need to consider how MF fields "pack" specific *synchronous dynamics* "vertically" ranging across these different domains, just as we have considered *diachronic dynamical fields* spanning "horizontally" within a single domain.

If we are able to accomplish this reframing of our perceptual and cognitive habits in order to recognize Mind Force, we might see how it forms a dynamical "glue" ensuring attractions in bodies, minds and social ensembles and the cohesion of our inner and outer bio-psychosocial realities. A definition of Mind Force would be as *the dynamical hyperstructure formed by networks of synchronized oscillators coupled in fields spanning trough heterogeneous domains* (Orsucci, 2009).

In conclusion, Mind Force theory is based on the integration of 4 main pillar-theories:

- *Complexity theory*
- *Synchronization theory*
- *Network theory*
- *Field theory*

Fig 5 here, the four pillars of Mind Force

Within the population of bio-psychosocial oscillators some act as master hubs, others as slave or free (Nicolis & Prigogine, 1977; Kauffman, 1993; 1995; 2000; Kelso, 1995) nodes within the MF hyperstructure dynamics. Waves of massive and heterogeneous transient entrainment would form attractors and fields. These waves of massive synchronizations propagate through different media, domains and scales. A logical consequence is that MF transient or steady fields interfere and interact, producing MF resultants, forming MF dynamical landscapes.

In conclusion, to use an expression coined by Douglas Hofstadter (2007: 39) Mind Force is the result of "the causal potency of collective phenomena" and patterns.

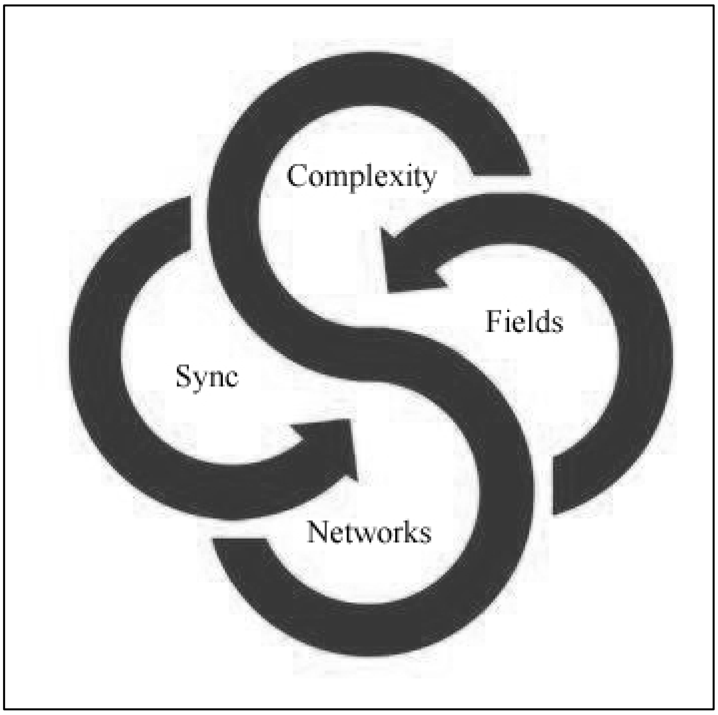

Fig 5, the four pillars of Mind Force

8. Modeling.

The basic modeling for Mind Force is grounded in the network of oscillators' dynamics. To start, a pair of oscillators interacting through phase differences satisfies equations of the form

$$\theta_1' = \omega_1 + H_1(\theta_2 - \theta_1)\ ,$$

$$\theta_2' = \omega_2 + H_2(\theta_1 - \theta_2)\ .$$

Here θ_i are the phases of the oscillators, ω_i are the frequencies of the uncoupled oscillators, and H_i are smooth 2π-periodic functions of the phase differences.

From a more abstract viewpoint, synchronization has been identified as a generic form of collective behavior in ensembles of dynamical systems with long range coupling. Several models that capture the essence of synchronization phenomena have been thoroughly studied over the last few decades. Kuramoto (1984) for instance, has analyzed an ensemble of N coupled phase oscillators, governed by the equations

$$\dot{\phi}_i(t) = \Omega_i + \frac{\varepsilon}{N}\sum_{j=1}^{N}\sin(\phi_j - \phi_i)\,,$$

$i = 1,\ldots,N$, where $\varepsilon > 0$ is the strength of coupling. In the absence of coupling, $\varepsilon = 0$, each oscillator *i* performs a uniform angular motion with its natural frequency Ω_i. For $\varepsilon \neq 0$, the oscillators are globally coupled in the sense that the strength of the pair interaction does not depend on their relative position, but only on their relative state. In other words, each oscillator interacts with the rest of the system through global averages only.

Kuramoto has shown that, in the limit $N \rightarrow \infty$, there exists a critical value ε_c of the coupling intensity such that, for $\varepsilon > \varepsilon_c$, a subensemble of oscillators becomes entrained in periodic orbits with the same frequency, whereas the other oscillators remain unsynchronized.

A great deal of attention has been paid to the synchronization of ensembles formed by identical elements, especially, in the case where the individual dynamics is chaotic. Both continuous and discrete-time dynamics have been considered. Kaneko (1994) has introduced globally coupled chaotic maps as a mean-field model of lattice maps, which are extensively used to model complex extended systems (Kaneko, 1993).

For an ensemble of *N* maps whose individual dynamics is governed by the equation $w\,(t + 1) = f\,[w(t)]$, global coupling is introduced as

$$\begin{aligned}\mathrm{w}_i(t+1) &= (1-\varepsilon)\mathbf{f}[\mathrm{w}_i(t)] + \frac{\varepsilon}{N}\sum_{j=1}^{N}\mathbf{f}[\mathrm{w}_j(t)] \\ &= (1-\varepsilon)\mathbf{f}[\mathrm{w}_i(t)] + \varepsilon\overline{\mathbf{f}(\mathrm{w})}\,,\end{aligned}$$

$i = 1,\dots,N$, with $\varepsilon \in [0, 1]$. While for $\varepsilon = 0$ the elements evolve independently, for $\varepsilon = 1$ they become fully synchronized after the first time step.

Full synchronization is understood here as a situation where the individual states of all the elements in the ensemble coincide, i.e. where the trajectory of the system in phase space is restricted to the subspace $w_1 = w_2 = \dots = w_N$. In this situation, the evolution of all the elements coincides with that of an independent element. The state of full synchronization can be asymptotically approached as the system evolves even for $\varepsilon < 1$. It has been shown that, if the individual dynamics is chaotic, full synchronization is linearly stable for $\varepsilon > \varepsilon_c$, where the critical value ε_c is related to the maximal Lyapunov exponent λ_M of the individual dynamics, as $\varepsilon_c = 1 - \exp(-\lambda_M)$. For nonchaotic individual dynamics where $\lambda_M < 0$, full synchronization is a stable state for any $\varepsilon > 0$. The connection between ε_c and λ_M makes it clear that the transition to full synchronization in chaotic systems, which has the character of a critical phenomenon, results from the competition between the stabilizing effect of global coupling and the inherent instability of chaotic orbits. Note carefully that the critical value ε_c does not depend on N, so that the synchronization threshold is the same for any size of the coupled ensemble.

For coupling strengths just below ε_c the system evolves asymptotically to a state of partial synchronization in the form of *clustering*, where the elements become divided into groups (Kaneko, 1989). Within each cluster the elements are fully synchronized but different clusters have different trajectories.

For large systems, the dynamics in the clustering regime is highly multistable and exhibits glassy-like features (Crisanti et al., 1996; Manrubia & Mikhailov, 2001). In contrast with the critical value ε_c, the stability properties of the clustering regime are strongly dependent on the system size (Abramson, 2000).

Cross-Coupled Extended Systems provide a versatile collection of models for a wide class of complex natural phenomena, ranging from pattern formation in physicochemical reactions, to biological morphogenesis, to evolutionary processes.

It is therefore interesting to consider how these systems behave under the effect of mutual interactions and, in particular, study the synchronization properties of their coevolution when they are mutually coupled by algorithms similar to the scheme of coupling already presented (Zanette & Morelli, 2003).

P.M. Gleiser and D.H. Zanette (2006) analyzed the interplay of synchronization and structure evolution in an evolving network of phase oscillators. An initially random network is adaptively rewired according to the dynamical coherence of the oscillators, in order to enhance their mutual synchronization. They showed that the evolving network reaches a small-world structure. Its clustering coefficient attains a maximum for an intermediate intensity of the coupling between oscillators, where a rich diversity of synchronized oscillator groups is observed. In the stationary state, these synchronized groups are directly associated with network clusters.

Their model consists of an ensemble of N coupled phase oscillators, whose individual evolution is given by

$$\dot{\phi}_i = \omega_i + \frac{r}{M_i} \sum_{j=1}^{N} W_{ij} \sin(\phi_j - \phi_i),$$

$i = 1,\ldots,N,$ where ω_i is the natural frequency of oscillator I and r is the coupling strength. The weights W_{ij} define the adjacency matrix of the interaction network: $W_{ij} = 1$ if oscillator i interacts with oscillator j, and 0 otherwise. The number of neighbors of oscillator i is $M_{ij} = \sum_j W_{ij}$. The adjacent matrix is symmetric, $W_{ij} = W_{ji}$, so that the network is a non directed graph.

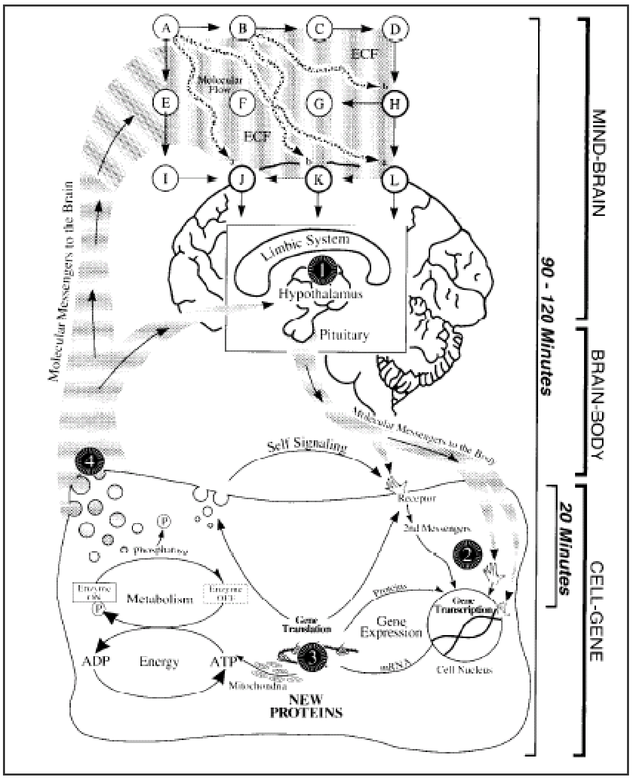

Fig. 1, Heterogeneous couplings in hyperstructure (Rossi, 1996)

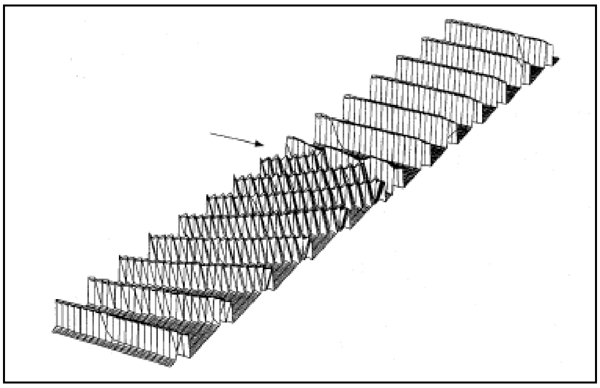

Fig. 2, Start of synchrony and travelling waves in relaxation oscillators (Somers & Kopell, 1995)